Hale School **Physics 3A 2011**

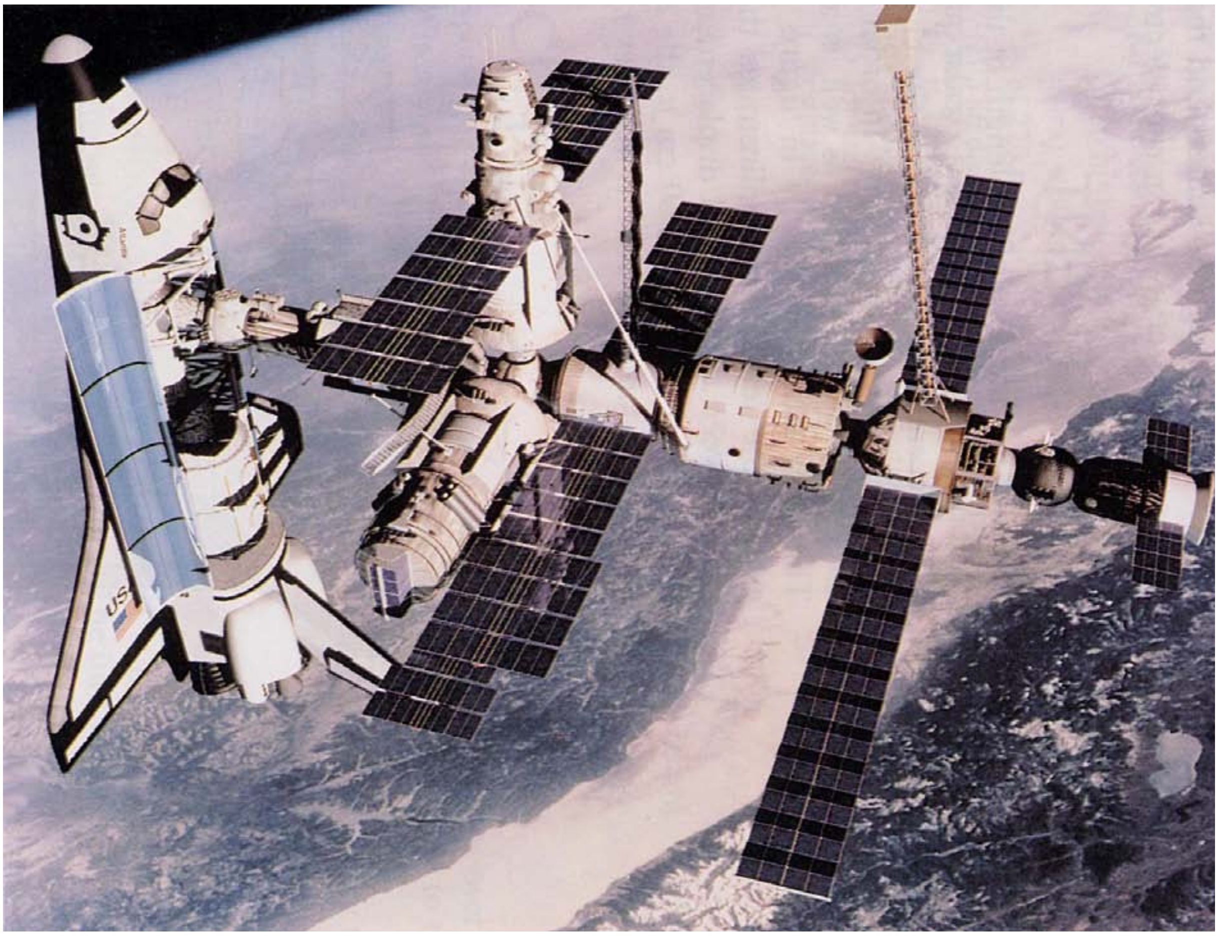
Motion and Forces in a Gravitational Field

Year 12 Study Notes

Name:

Teacher:

Set:



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Kinematics

Speed is the distance moved by a body per unit time.

Average speed is given by: $\bar{\upsilon} = \frac{d}{t}$

Velocity is defined as the change in a body's displacement per unit time.

Average velocity is given by: $\bar{v} = \frac{s}{t}$ $\bar{v} = \frac{u + v}{2}$

Velocity is a vector quantity and thus has direction.

Acceleration is the time rate change in a body's velocity $\vec{a} = \frac{\Delta \vec{v}}{t} = \frac{\vec{v} - \vec{u}}{t}$

Acceleration is a vector quantity and thus has direction.

Acceleration may result from:

- a change in the bodies speed
- a change in the bodies direction
- a change in both speed and direction

Negative Acceleration: If a body's velocity decreases with time then it undergoes a negative acceleration or is said to <u>decelerate.</u>

Equations of Uniform Motion

1st Equation of Motion	2nd Equation of Motion	3rd Equation of Motion
$v = u + at$	$v^2 = u^2 + 2as$	$s = ut + \frac{1}{2}at^2$

<u>Direction and the Equations of Motion:</u>

Note that equations of motion deal with vector quantities and direction of the quantity must be taken into account.

The following procedures must be carried out when dealing with vector quantities:

- assign a direction as positive;
- any vector in that direction is given a positive value;
- any vector in the opposite direction is given a negative value.

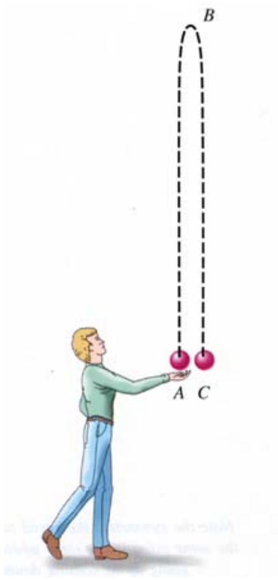
Acceleration Due To Gravity

Equations Of Motion For Falling Bodies:

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs$$



<u>Points to Consider:</u>

When an object is projected upwards close to the earth's surface :

- the acceleration due to gravity "g" always acts down;
- its velocity is zero at its maximum height;
- its displacement is zero when it returns to its initial position;
- its speed at a given height is the same for the upward and downward paths.

For all problems involving acceleration due to gravity, it is essential that the direction of all vector quantities involved be taken into account.

Dynamics

Newton's 1st Law of Motion:

A body will remain at rest or continue in its state of uniform motion unless acted upon by a net external force.

In other words: a body's velocity remains unchanged unless acted upon by a **net** external force.

Inertia: Inertia is the tendency of a body to remain at rest or continue with constant velocity.

The greater the mass of the body, the greater its inertia.

Momentum: Momentum is defined as the product of a body's mass and its velocity.

Thus momentum = mass x velocity $\boxed{\vec{p} = m\vec{v}}$

Momentum is a vector quantity. The body's momentum is in the same direction as its velocity.

Change in Momentum:

Change in momentum is the vector difference between the bodies final and initial momenta.

Change in Momentum = final momentum – initial momentum $\boxed{\Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}}$

Note the direction of the change in momentum is equal to the direction of the change in velocity $\Delta\vec{v}$.

$$\Delta\vec{p} = m\vec{v} - m\vec{u} = m(\vec{v} - \vec{u})$$

Since $\Delta\vec{v}$ determines the direction of the acceleration $\vec{a}$, this is also the direction of the net force on the body.

$$\boxed{\therefore \Delta\vec{p} = m\Delta\vec{v}}$$

Newton's 2nd Law:

When a net external force is applied to a body it produces a change in the body's momentum.

The rate of change in the momentum of the body is directly proportional to the applied net force and in the direction of the force. $F \propto \frac{\Delta p}{t} \quad \therefore \; F = k\frac{\Delta p}{t}$

Thus $\boxed{\vec{F} = \frac{\Delta \vec{p}}{t}}$ Since $F = \frac{\Delta p}{t}$ and $\Delta p = m(v-u)$

$$F = m\frac{v-u}{t} = ma$$

$$\text{ie} \quad \boxed{F = ma}$$

When a net external force is applied to a body it produces an acceleration in the direction of the force.

The acceleration produced is directly proportional to the applied force and indirectly proportional to the mass of the body.

Weight: The gravitational force exerted by gravity on a body due to its mass is termed weight.

The weight of a body is given by: $\boxed{W = mg}$

Newton's Third Law: To every action there is an equal and opposite reaction.

Thus forces always act in pairs (action and reaction).

Note: The action and reaction forces act on different bodies and thus do not cancel each other.

Impulse: The product of a force and the time for which it acts is termed impulse "J".

Impulse = force x time $\boxed{\vec{J} = \vec{F}t}$

Impulse is a vector and has the same direction as $\vec{F}$.

From Newton's 2nd Law: $\vec{F} = \frac{\Delta \vec{p}}{t}$ or $\boxed{\vec{F}t = \Delta \vec{p}}$

Thus the impulse of a body can be directly determined by its change in momentum.

Conservation of Linear Momentum:

When two or more bodies interact, the sum of their momenta before interaction is equal to the sum of their momenta after interaction.

$$\text{ie} \; \Sigma \vec{p}_{\text{before}} = \Sigma \vec{p}_{\text{after}}$$

The total momentum of a closed system will remain constant.

This equation is true for all interactions ie collisions and explosions.

Work: Work is done when the point of application of a force is moved in the direction of the force.

$$W = F s$$

Energy: Energy may be defined as the capacity of a body to do work.

Kinetic Energy: The energy possessed by a body due to its motion is termed kinetic energy.

$$E_K = \frac{1}{2} m v^2$$

Potential Energy: The energy of a body due to its position or state is termed its potential energy.

$$E_p = m g h$$

Gravitational Potential Energy:

When a body is lifted up to a level above the Earth's surface its potential energy increases as a result of its change in position relative to the Earth.

Conservation of Energy: Energy can neither be created nor destroyed.

$$E_T = E_K + E_P$$

Thus the total energy content of a system remains constant. However energy may be transformed from one form to another. Since energy is conserved the total energy of the system does not change.

In practice when energy is transferred or transformed (ie work is done), a proportion of the energy is dissipated and converted into non useable forms.

Power: The amount of work done per unit time is termed power.

$$P = \frac{W}{t}$$

Power and Velocity: Since $P = \frac{W}{t}$ and $W = F s$

$P = \frac{F s}{t}$ Thus $P = F v$

Exercise Set 1: Unit 2A Revision

1. A camera operator accidentally drops a camera from a stationary helicopter at a height of 122.5 m.
   a) Calculate the time of the flight of the camera.
   b) What is the speed of the camera just before it hits the ground?

2. A skydiver is free falling at a height of 980 m. Her speed is measured as 40 ms^{-1}. At this stage she opens her parachute which alters her speed to a constant value of 5.0ms^{-1}. At the instant she opens her parachute, she drops a camera she was holding. How long is it between when the camera hits the ground and when the parachutist lands?

3. A 90.0 kg man stands in a lift. Find the force which the floor of the lift exerts on the man:
   a) when the lift has an upward acceleration of 2.00 ms^{-2};
   b) when the lift is rising at constant speed;
   c) when the lift has a downward acceleration of 2.00 ms^{-2}.

4. In a game of snooker a player hits a 0.20 kg billiard bail with the cue, exerting an average force of 40 N south on the ball for 12 millisecond.

a) What is the impulse of the force exerted on the ball?

b) What is the change in momentum of the ball?

c) With what velocity does the ball leave the cue?

5. A 75 g golf ball is struck with a golf club. The club and the ball are in contact for 0.100 s during which time the ball accelerates to 50.0 ms^{-1}.

a) What is the impulse acting here?

b) What force acts on the ball?

c) What force acts on the club?

6. Two gliders A and B collide on an air track. A has a mass of 75 g and B 100 g. They travel towards each other, each moving at 1.0 ms^{-1}, then collide and stick together. What is their final velocity?

7. A body of mass 0.36 kg is projected horizontally with a speed of 8.0 ms^{-1}, from the top of a tower AD which is 45 m above the level ground DC. It moves on a parabolic path and strikes the ground at C as shown in Figure 19.1. B is a point on the path of the body 32 m above the ground. Ground level is taken as the zero level of gravitational potential energy.

a) What is the total energy of the body at A immediately after launch?

b) How much potential energy has it lost in moving from A to B?

c) What is its kinetic energy at B?

d) What is its speed at B?

e) What is its kinetic energy just before it strikes the ground at C?

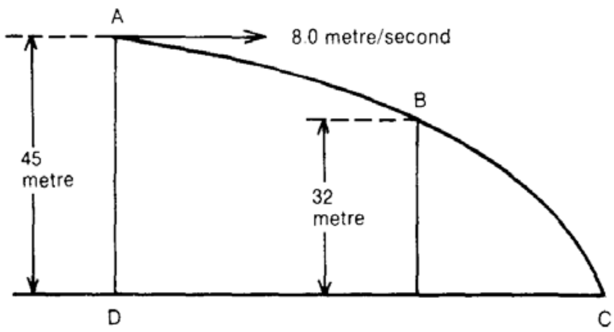


Fig. 19.1

8. A car of mass 2000 kg travels up a slope of 30.0° at a uniform speed of 15.0 m s^{-1}. If the frictional resistance is 10% of the weight of the car, at what power is the engine working?

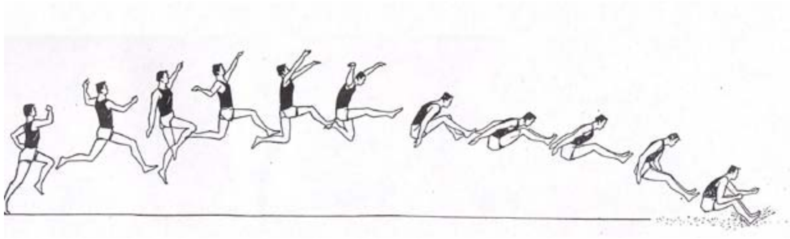
Answers

1. a) 5 s b) 49 ms^{-1}
2. 185s
3. a) 1062 b) 882N c) 702N
4. a) 0.48 N s south b) 0.48 kgms^{-1} south c) 2.4 ms^{-1} south
5. a) 3.75 Ns in direction of the club b) 37.5 N in direction of the club
   c) 37.5 N in opposite direction to the club
6. 0.143 ms^{-1} same direction as B's initial motion
7. a) 2.0 x 10^{2} J b) C c) 78 J d) 1.2 X 10^{2} J e) 14 ms^{-1}
8. 1.76 X 10^{5} W

Projectile Motion

An object thrown into the earth's gravity field experiences two forces

1. the initial accelerating force
2. the force of the gravity field.



The object's <u>trajectory</u> is **parabolic**, and depends on

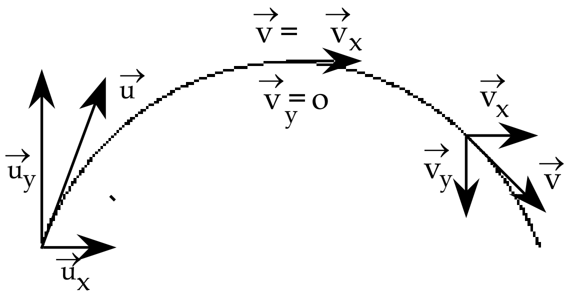
a) its initial velocity (including direction),

b) the strength of the gravity field

<u>In all cases</u> a projected object's velocity can be resolved into two components,

i.e. $\vec{v} = \vec{v}_x + \vec{v}_y$

where v_x is the horizontal component and v_y the vertical component.



<u>Notice :</u>

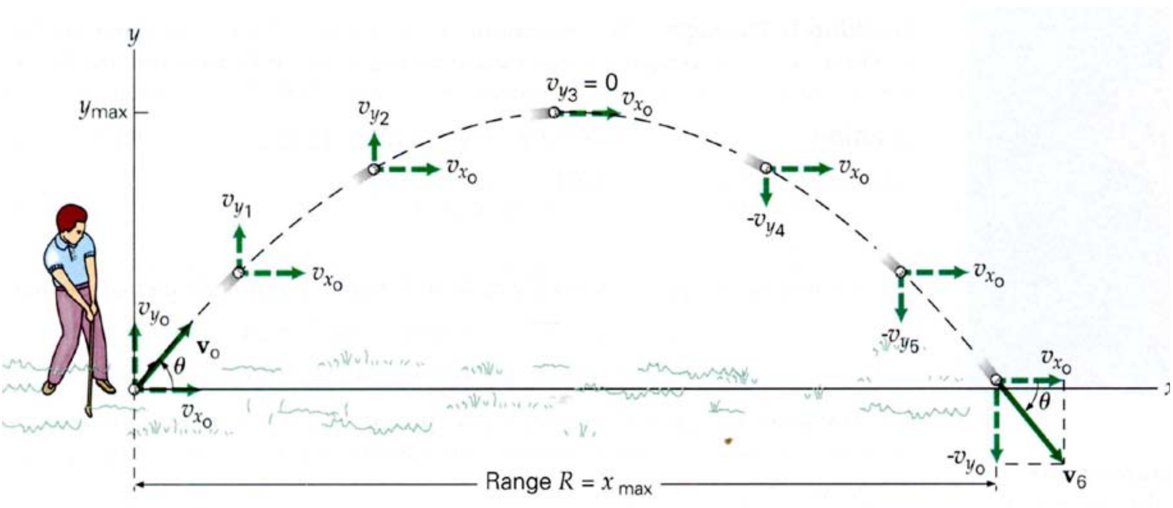
1. $\vec{v}_x$ (the horizontal component) remains unchanged;

2. $\vec{v}_y$ (the vertical component) is changed throughout by the acceleration due to gravity;

3. $\vec{v}$ (the instantaneous velocity) at any time t is found by the vector addition of $\vec{v}_x + \vec{v}_y$;

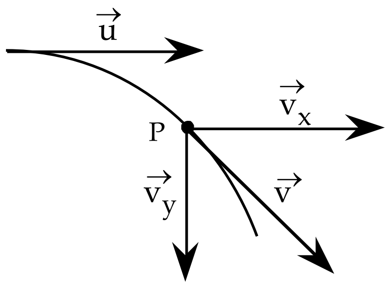
4. $\vec{v}_x$ and $\vec{v}_y$ are totally independent of each other;

5. If the data is expressed in cartesian components (as above) all straight line motion formula can be used. e.g. $s_y = u_y t + \frac{1}{2} a_y t^2$



Horizontal Projection

Consider a body projected horizontally with initial velocity u.



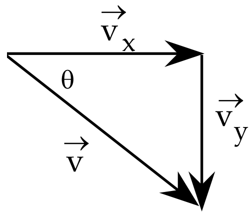
At P :

$v_x = u_x = u$

$v_y = u_y + gt$

but $u_y = 0 \text{ ms}^{-1}$

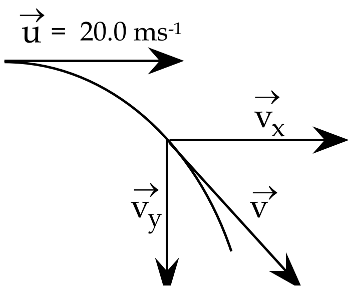
$\therefore v_y = gt$



Note:

- that the components are mutually at right angles and therefore are independent.
- The projectile's velocity is the vector sum of the horizontal and vertical components.

Example 1: A ball is thrown horizontally at 20.0 ms^{-1}. After 5.00 s determine:
(i) its horizontal velocity (ii) its vertical velocity (iii) total resultant velocity.

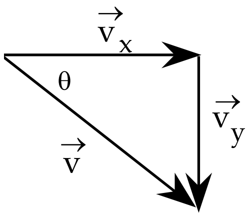


(i) $v_x = u_x = u = 20.0 \text{ ms}^{-1}$

(ii) $u_y = 0.00 \text{ ms}^{-1}$

$v_y = u_y + gt = 0 + 9.8 \times 5$

$= \underline{49.0 \text{ ms}^{-1}}$



$\vec{v} = \vec{v}_x + \vec{v}_y$

$v^2 = v_x^2 + v_y^2$

$= 20^2 + 49^2$

$= 2801$

$v = \underline{52.9 \text{ ms}^{-1}}$

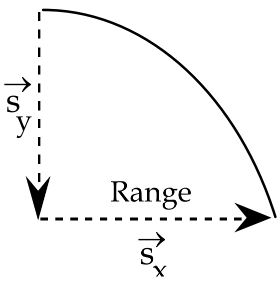
$\tan\theta = \frac{49}{20}$

$= 2.45$

$\therefore \theta = \underline{67.8^o}$ to the horizontal.

$\vec{v}$ = **52.9 ms^{-1} at 67.8^o to the horizontal.**

The displacement of a projectile has horizontal and vertical components that can be calculated.



At time t :

$s_x = v_x t = u\,t$

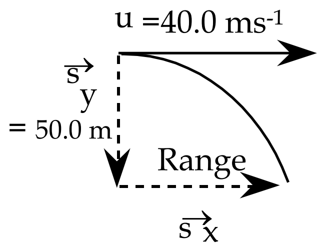
$s_y = u_y t + \frac{1}{2} a_y t^2$

Note for a horizontal projectile $u_y = 0$

$$s_Y = \frac{1}{2} g t^2$$

Example 2: A ball is thrown at 40.0 ms^{-1} horizontally from a cliff 50.0 m high. Find the horizontal range.

u_x = 40.0 ms^{-1}
s_y = 50.0 m



$$s_Y = \frac{1}{2} g t^2$$
$$t^2 = \frac{2 s_y}{g}$$
$$t^2 = \frac{2 \times 50}{9.8}$$
$$t = 3.19\,s$$

$s_x = u_x t$

= 40 x 3.19

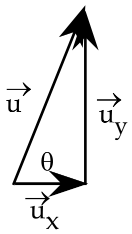
= <u>128 m</u>

Range = <u>128 m</u>

Problem 1: A bullet is fired from a gun horizontally at 500.0 ms^{-1}. Determine its range if it was fired from a point 2.00 m above the ground.

Oblique Projections

Consider a body projected at an angle θ to the horizontal with a velocity $\vec{u}$.



u_x = u × cosθ
u_y = u × sinθ

The initial velocity of the body is resolved into its horizontal and vertical components, $\vec{u}_x$ and $\vec{u}_y$

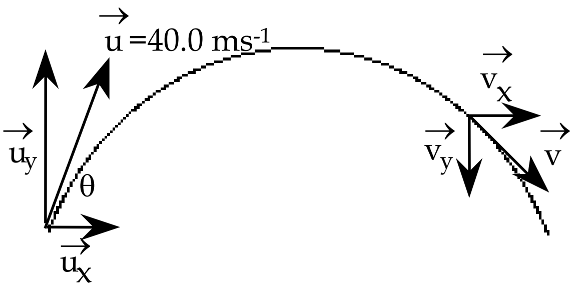
At any time t during the flight of the body:

(i) the horizontal component of the projectiles velocity $v_x = u_x = u \times \cos\theta$

(ii) the vertical component is given by: $v_y = u_y + gt = u \times \sin\theta + gt$

[Note the sign convention for vertical motion must be observed.]

Example 3: A stone is thrown at 60.0^0 to the horizontal at 40.0 ms^{-1}.
Calculate the velocity of the stone 5.00 s after it is thrown.



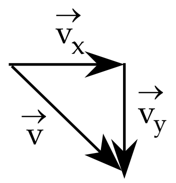
(i) $v_x = u_x = u \times \cos\theta$

$= 40 \cos 60 = \underline{20.0\ ms^{-1}}$

(ii) $v_y = u_y + gt = u \times \sin\theta + gt$

$= 40 \times \sin 60 + -9.8 \times 5$

$= -\underline{14.36\ ms^{-1}}$



$\vec{v} = \vec{v}_x + \vec{v}_y$

$v^2 = v_x^{\ 2} + v_y^{\ 2} = 20^2 + (-14.36)^2$

v = $\underline{24.6\ ms^{-1}}$

$$\tan\theta = \frac{v_y}{v_x} = \left|\frac{-14.36}{20}\right|$$

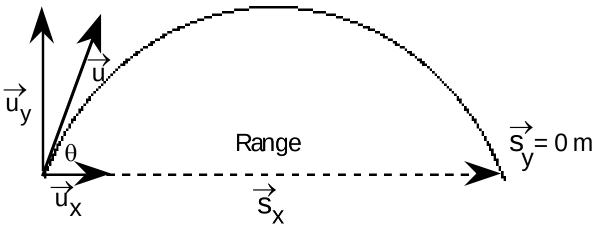
$$\therefore \theta = 35.7^o$$

$\therefore \vec{v} =$ **24.6ms^{-1} at 35.7° below the horizontal**

Problem 2: A golf ball is hit at 100.0 ms^{-1} at 40.0^0 to the horizontal.
Determine the horizontal and vertical components of its velocity and hence its resultant velocity 3.00 seconds later.

Displacement

The displacement may be determined by considering the vertical and horizontal components.



The horizontal displacement at any time t is given by: $s_x = u_x t = u \times \cos\theta \times t$

The vertical displacement is given by:

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

Horizontal range is the horizontal displacement to the impact position of the projectile.

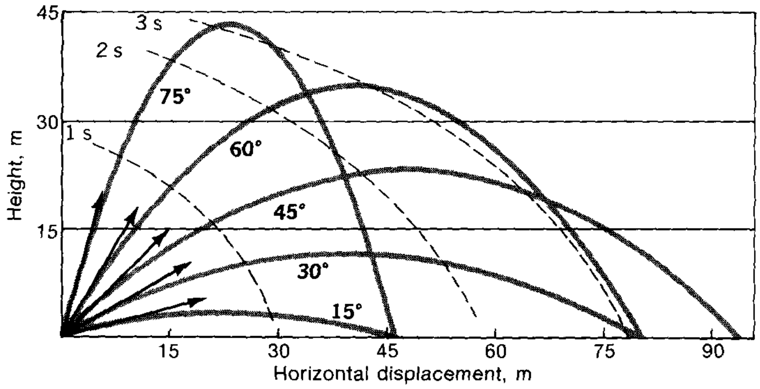
Range depends on the size of both u and θ.

There are 2 angles for which a particular range is obtained.
They add up to 90°; e.g. 30° and 60° or 75° and 15°.

The diagram shows the approximate paths of balls projected with the same initial speed ($30 ms^{-1}$)

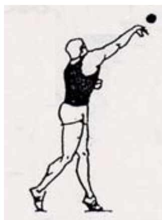
Dashed lines show the positions of the balls after 1, 2 and 3 seconds.

Launched from the ground, an elevation of 45° produces the maximum horizontal range.



Due to the practicality of launching above the ground, successful horizontal range competitions (eg Shot put) are the result of a compromise between attaining the maximum horizontal velocity and optimal flight time.

[draw trajectories to illustrate this]

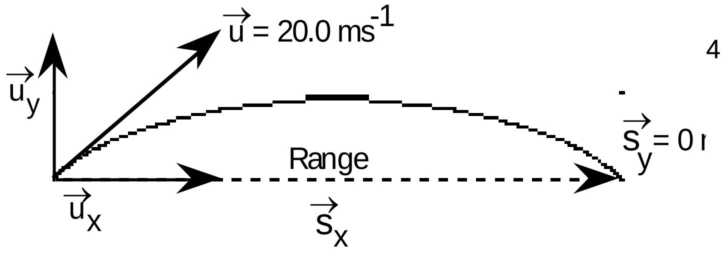


Example 4: A ball is thrown at $20.0 ms^{-1}$ at 40.0^0 to the horizontal. Find the horizontal range.

s_y = 0.00m

u = $20.0 ms^{-1}$ at 40.0° to horizontal

Let up be positive



$$s_y = u_y t + \frac{1}{2} g t^2$$

$$= u \times \sin\theta \, t + \frac{1}{2} g t^2$$

$$0 = 20 \times \sin 40^0 t - 4.9 t^2$$

$$4.9 t = \sin 40^0 \times 20$$

t = 2.62 s

$s_x = u \times \cos\theta \, t$

$= 20 \times \cos 40^0 \times 2.62$

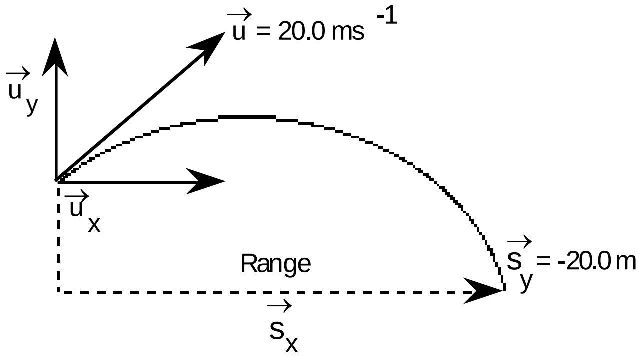
So, Range = 40.2 m

Example 5: If the ball in example 1 is thrown from a ledge 20.0 m above the ground, find the horizontal range.

$s_y = -20.0$ m

u = 20.0 ms^{-1} at 40° to horizontal

let up be positive; g= - 9.8 ms^{-2}



$$s_y = u_y t + \frac{1}{2} g t^2$$

$$= u \times \sin 40^0\ t + \frac{1}{2}\ g t^2$$

$$-20 = \sin 40^0 \times 20t - 4.9t^2$$

$$4.9\ t^2 - 12.85\ t - 20 = 0$$

$$t = \frac{12.85 \pm [12.85^2 - 4 \times 4.9 \times (-20)]^{\frac{1}{2}}}{2 \times 4.9}$$

$$t = \frac{12.85 \pm 23.6}{9.8} = \underline{3.72\ s}$$

$$s_x = u_x t = u \times \cos 40^0\ t$$

$$= 20 \times \cos 40^0 \times 3.72$$

So, Range = <u>57.0 m</u>

Example 6: A golfer must hit a ball 120.0 m to a green which is elevated 10.0 m above the tee. If the golfer hits the ball at 40.0^o to the horizontal, with what velocity must he hit the ball?

s_x = 120.0 m

let up be positive

s_y = 10.0 m

g = -9.8ms^{-2}

$s_x = u_x t = u \times \cos 40^0\ t$

$$t = \frac{s_x}{u \cos 40}$$

$$= \frac{120}{u \cos 40}$$

$$= \frac{156.6}{u}$$

$$s_y = u_y t + \frac{1}{2}\ g t^2$$

$$= u \times \sin 40\ t + \frac{1}{2}\ g t^2$$

$$10 = u \times \sin 40 \frac{156.6}{u} - 4.9 \left(\frac{156.6}{u}\right)^2$$

$$100.3 - 10 = \frac{4.9 \times 156.6^2}{u^2}$$

$$u^2 = \frac{4.9 \times 156.6^2}{90.3}$$

$$\underline{u = 36.5\ ms^{-1}}$$

Problem 3: A basketball is projected upwards at 60.0^0 to the horizontal at 6.25 ms^{-1} passing through a ring 1.00 m above the point from which the ball was released.
Find the distance the ring is from the player.

Maximum Range over level ground

Note: This applies only where <u>vertical displacement of the projectile is zero</u>.
Consider a projectile given an initial velocity $\vec{u}$ at θ^0 to the horizontal.

$$s_y = u_y t + \frac{1}{2} gt^2 = u \times \sin\theta \times t + \frac{1}{2} gt^2$$

At impact $\vec{s}_y = 0$ m
(initial height and impact height must be the same).

$$0 = u \times \sin\theta \times t + \frac{1}{2} gt^2$$

$$\frac{1}{2} gt = -u \times \sin\theta$$

$$t = \frac{-2u.\sin q}{g} \quad \text{...... [1]}$$

$$s_x = u_x t = u \times \cos\theta \times t \quad \text{.......[2]}$$

Sub. [1] into [2]

$$s_x = \frac{-2\, u \sin\theta \times \cos\theta\, u}{g}$$

$$s_x = \frac{-2 \times \sin\theta \times \cos\theta \times u^2}{g}$$

but $\sin 2\theta = 2 \sin\theta \cos\theta$

and g is always negative with respect to u,

$$\boxed{s_x = \frac{u^2 \sin 2q}{g}}$$

Now for maximum range sin 2θ must be a maximum, i.e. $2\theta = 90^o$

<u>$\theta = 45^o$</u>

Notes on projectiles:

When a projectile reaches maximum height its vertical component is zero. $v_y = 0.00\ ms^{-1}$

When a projectile is fired at ground level and impacting at ground level :

- the vertical displacement at impact, $s_y = 0.00$ m
- the time taken to reach maximum height is half the total flight time
- the speed at impact equals the initial speed. [Note velocities are different!]

Problem 4: During a round of golf, a player needs to land the ball on the green 150.0 m away. What possible angles to the horizontal does the player need to hit the ball if the speed of the ball off the club is $50.0\ ms^{-1}$ and the ball is to land on the green?

Air Effects on Projectile Motion

As a projectile moves through the air it experiences a retardation (opposing force).

The retardation is due to:

1. inertial effects, i.e. work is done on air particles to give them kinetic energy as they are pushed out;

2. viscous drag, i.e. the movement of air particles over the projectile's surface producing a frictional effect.

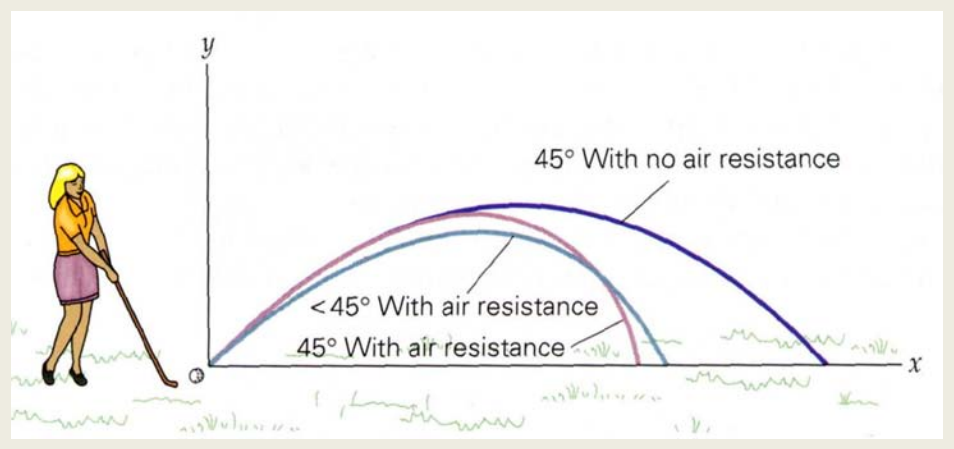
The effect of each is dependent upon the velocity of the projectile.

The higher the velocity, the greater the effect of viscous drag.

The overall effect of air resistance is to reduce the maximum vertical height and the range of the projectile.

The actual path taken by the projectile will depend on a number of factors including its mass, size shape and velocity.

If the projectile is spinning this may also affect the resultant path.



Exercise Set 2: Projectile Motion

Ignore the effects of air resistance and friction unless otherwise indicated.

1. Calculate the vertical and horizontal components of the following velocities:
   a) $10ms^{-1}$ at 30^0 from the horizontal. b) $70ms^{-1}$ at 45^0 from the horizontal.
   c) 36 kmh^{-1}, vertically up.

2. A stone is released from the top of a 122.5 m cliff. How long will it take to reach the ground:
   a) if it's released from rest? b) if it's thrown horizontally at 20 ms^{-1}?

3. Calculate the final velocity of the stone in both situations described in question 2.

4. An object is projected from a cliff at 5.0 ms^{-1} horizontally. It is carefully timed and found to hit the ground after 4.0s.
   a) What is the height of the cliff?
   b) What is the range of the object's motion?
   c) What is the final velocity of the object?

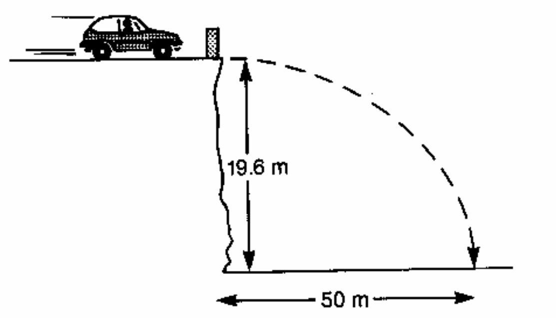
5. A golf ball is hit with a velocity of 40 ms^{-1} at 40° to the horizontal.
   a) Calculate the vertical and horizontal components of its initial velocity.
   b) How high will the ball travel?
   c) How long will the ball be in the air?
   d) What is the range of the ball? (Ignore the fact that it will bounce on landing.)

6. A light plane flies at 144 kmh^{-1} at a height of 490 m over a desert region. The pilot is to drop supplies for some people stranded below. She spots them and flies in to make the drop. How far before their camp should she release the parcels?

7. A rifle fires a bullet with a muzzle speed of 300 ms^{-1}. If the rifle sights are aligned with the barrel, find how far above a bull's-eye 100 m away the rifle should be aimed to hit the centre. (The usual practice is to adjust the sights to allow for the fall of the bullet for some prescribed target distance.)

8. A car crashes through a barrier, drives over a 19.6 m high cliff that was just the other side of the barrier, and lands 50.0 m away from its base.

Was the driver breaking the 80.0 kmh^{-1} speed limit at the time of crashing through the barrier?

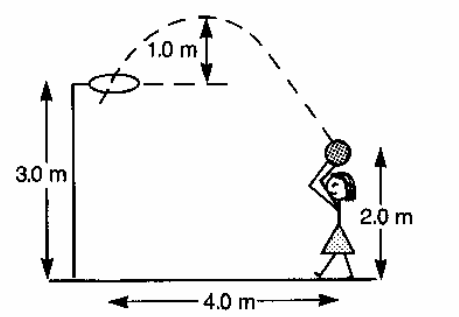


9. You are an expert physics student and golfer. On the eighteenth hole you must sink a small chip to win the Physics Championship. The hole is 25.0 m away and you use your nine iron which will project the ball at 60.0° to the horizontal.

Assuming the acceleration due to gravity is 10.0 ms^{-2}, determine the initial speed that must be given to the ball to land it in the hole 'on the fly'.

10. A netball player shoots for goal. She is standing 4.00m from the goal which is 3.00m above ground level.

The ball is released from a height of 2.00m above ground level and at its maximum height it is 1.00m above the goal.



a) What is the ball's acceleration at its maximum height?
b) If the ball is thrown with a velocity v at an angle θ to the horizontal, which of the following is an expression for its speed at the maximum height?
I) v II) v×sinθ III) v×cosθ IV) tanθ V) 0 - give a reason for your answer.
c) What is the value of the vertical component of the initial velocity?
d) How long after leaving the player's hands does the ball reach its maximum height?
e) After reaching the maximum height, how much longer does it take for the ball to reach the goal ring?
f) From the point of release, how long does it take for the ball to pass through the goal ring?
g) Determine the initial horizontal component of the velocity of the ball.
h) Using your answer to g), find the velocity with which the ball was thrown.

11. A stone (A) is thrown off a cliff horizontally with a velocity of 25.0 ms^{-1} from a height of 78.4 m above sea level. A second stone (B) is thrown from the same height at the same instant but with a velocity of 19.6 ms^{-1} at an angle of 30.0° up from the horizontal. Assume there are no waves on the sea.

a) How long will stone A take to hit the sea?
b) How far horizontally will stone A travel?
c) How long will it take for stone B to reach its maximum height?
d) How high above the point from which it is thrown will stone B go?
e) How long will it take stone B to fall from its maximum height to the sea?
f) Which stone will travel further horizontally and by how much?
g) What is the velocity of stone A after 3.0 s?

12. At the Superbowl kick-off a football is given a velocity of 25.0 ms^{-1} at an angle of 53.1^0 with the horizontal. Find:
a) how long the ball is in the air,
b) how high the ball goes,
c) how far from the kick-off point the ball lands, and
d) the magnitude of the velocity of the ball 3.00 s after it is kicked.

13. A boy kicks a soccer ball off the ground, giving it a speed of 18.0 ms^{-1} at an angle of 53.1° with the horizontal. Find:
a) how long the ball is in the air,
b) how high it goes, and
c) the horizontal distance it traverses before it strikes the ground.

14. A golf ball is driven horizontally with a speed of 40.0 ms^{-1} from a cliff 25.6 m high and it lands on a level plain below.
a) How long is the ball in the air?
b) How far from the base of the cliff does it strike the ground?

15. A baseball is batted with a speed 40.0 ms^{-1} at an angle of 36.9° with the ground. Find:
a) how long the ball is in the air,
b) how high the ball goes,
c) how far from the plate it is caught, and
d) the components of the velocity of the ball 3.00 s after it is hit.

16. A golf ball on a level fairway is given an initial velocity of 50.0 ms^{-1} at an angle of 35.5° with the horizontal. If air friction is neglected, find
a) how long the ball is in the air,
b) how high it goes and
c) how far away it lands.

17. A golfer chips a ball over a sand trap to an elevated green and 3.00 s after she hits the ball it lands on the green 50.0 m east and 12.0 m above her initial lie.
a) Find the components of the initial speed of the ball as it left the club.
b) Find the speed of the ball when it strikes the green.

18. A boy throws a stone toward the ocean with an initial velocity of 24.5 ms^{-1} 36.9° above the horizontal. The stone just clears a fence at the edge of a cliff; the height of the fence is luckily the same as the height at which the stone is released. The stone lands in the ocean 50.0 m below the top of the fence. Find
a) the distance from the boy to the fence,
b) the horizontal distance from the fence to the point at which the stone hits the ocean and
c) the speed of the stone the instant before it hits.

19. A golf ball is hit with a speed of 50.0 ms^{-1} angle of 30.0° with the horizontal from an elevated tee 12.0 m above the fairway.
a) How long is the ball in the air?
b) At what horizontal distance from the tee does the ball strike the ground?

20. A basketball player releases a ball 2.00 m above the floor when he is 10.0 m from the basket. The ball goes through the rim of the basket 3.00 m above the floor 1.50 s later. Find
a) the horizontal component of the initial velocity,
b) the vertical component of the initial velocity, and
c) the maximum height above the floor reached by the ball.

21. Find the minimum speed with which a cricket ball must be hit at an angle of 53.1° with the horizontal to leave the bat at a height of 1.20 m and clear a 10.0 m screen at a distance of 100 m from the batsman.

Answers

1. a) $v_V = 5ms^{-1}$ $v_h = 8.66ms^{-1}$; b) $v_V = 49.5\ ms^{-1}$ $v_h = 49.5\ ms^{-1}$ c) $v_V = 36\ kmh^{-1}$ $v_h = 0$

2. a) 5s b) 5s

3. a) 49 ms^{-1} down b) 53 ms^{-1} 67.8° down from horizontal

4. a) 78.4m b) 20m c) 39.5 ms^{-1} 82.7° down from the horizontal

5. a) $v_V = 25.7\ ms^{-1}$, $v_h = 30.6\ ms^{-1}$ b) 33.7m c) 5.25s d) 160.6 m

6. 400 m

7. 0.544 m

8. Yes; the car was travelling at 90 kmh^{-1}.

9. 17.0 ms^{-1} at 60° to the horizontal

10. a) 9.8 ms^{-2} down b) C c) 6.26 ms^{-1} up d) 0.64s e) 0.45 s f) 1.09 s
    g) 3.67 ms^{-1} h) 7.26 ms^{-1} at 59.6° up from the horizontal

11. a) 4.0s b) 100m c) 1.0s d) 4.9m e) 4.12 s f) A travels 13 m further
    g) 38.6 ms^{-1} at 49.6° down from horizontal.

12. a) 4.08 s b) 20.4 m c) 61.3 m d) 17.7 ms^{-1}, 32.1^0 below horizontal.

13. a) 2.94 s b) 10.6 m c) 31.7 m

14. a) 2.29 s b) 91.4 m

15. a) 4.90 s b) 29.4 m c) 157 m d) $v_{horiz} = 32.0\ ms^{-1}$, $v_{vert} = 5.38\ ms^{-1}$ down.

16. a) 5.93 s b) 43.0 m c) 241 m

17. a) $u_{horiz} = 16.7\ ms^{-1}$, $u_{vert} = 18.7\ ms^{-1}$; b) 19.8 ms^{-1}

18. a) 58.8 m b) 39.8 m c) 39.8 ms^{-1}

19. a) 5.54 s b) 240 m

20. a) 6.67 ms^{-1} b) 8.02 ms^{-1} c) 5.28 m

21. 33.1 ms^{-1}

Uniform Circular Motion

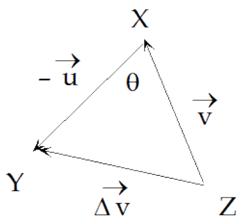
Centripetal Acceleration

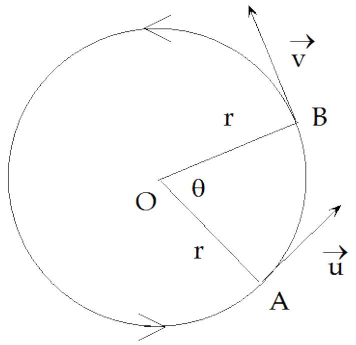
When a body moves in a circular path it is said to undergo circular motion.

Although the body may maintain a constant speed as it travels in a circle, its direction is continually changing. Thus the body is changing velocity and is accelerating.

Consider a mass on a string being swung in a circle of radius "r" and at a constant speed. In a small time period its velocity has changed from u to v.

Change in velocity $\Delta\vec{v} = \vec{v} - \vec{u}$





Let the velocity measured at two points A and B in the circular path centred at O be $\vec{u}$ and $\vec{v}$,

and the time taken between the measurements be Δt.

Consider ΔAOB and ΔXYZ in the diagrams showing the circular motion of the body and the vector diagram used to determine its change in velocity.

$$\Delta AOB = \Delta YXZ$$

Since XY = XZ □ΔYXZ is isosceles

and AO = BO , □□ΔAOB is isosceles

Thus ΔAOB and Δ□XYZ are similar triangles

$$\frac{AB}{OB} = \frac{YZ}{XZ}$$

$$\text{i.e.}\quad \frac{AB}{r} = \frac{\Delta v}{v} \quad \ldots \quad (1)$$

arc AB is the distance covered by the object in time Δt

$$\text{i.e. arc } AB = v\,\Delta t \quad \ldots \quad (2)$$

If Δt is small then chord AB = arc AB

Sub (2) in (1)

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Since the instantaneous acceleration

$$a = \frac{\Delta v}{\Delta t}$$

$$\text{i.e.}\quad \boxed{a = \frac{v^2}{r}}$$

As $\Delta t \to$ □0, $\theta \to 0^0$ and $\angle XZY = 90^{o}$, the direction of the <u>change in velocity</u> and thus the acceleration is towards the centre of the circle.

The acceleration necessary to keep the body moving in a circle with uniform speed is termed <u>centripetal acceleration</u>.

Summary

- The magnitude of the instantaneous acceleration of a body moving in a circular path is given by: $a = \frac{v^2}{r}$ (where v is the tangential speed of the body and r the radius of curvature).
- The direction of the centripetal acceleration at any given instant is towards the centre of the circle.

Problem 1: a) At what speed would a vehicle need to move across the surface of the Earth so that the centripetal acceleration is equal to the acceleration due to gravity?

b) What would the occupants of the vehicle experience if this were to happen? [Radius of the Earth $r_e = 6.38 \times 10^6$ m]

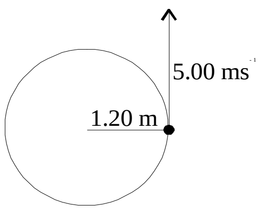
Period of Revolution

The period of revolution (T) is the time taken for one complete revolution of the body.

Velocity of a body moving in a circular path is $\dfrac{\text{circumference of circle}}{\text{time taken for one revolution}}$

$$v = \frac{2\pi r}{T}$$

Problem 2: A stone attached to the end of a piece of string 1.20 m long is whirled in a circular path at a speed of 5.00 ms^{-1}. Determine the period and frequency of revolution.



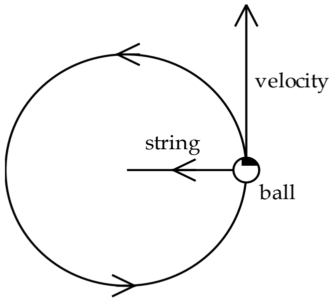
Centripetal Force

For a body to move in a circular path at constant speed, a net force directed towards the centre of the circle must be maintained. This force is termed the centripetal force F_c

A centripetal force is not an applied force, it is the **net force** acting on the body.

Centripetal force is always the **resultant** of one more applied forces that act on the body.

F_c = net force on body = ma but $a = \frac{v^2}{r}$ So, the centripetal force is given by: $F_c = \frac{mv^2}{r}$

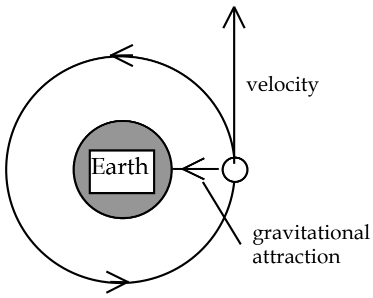


Examples and Applications of Centripetal Forces

1. **A ball connected to a string is whirled in a horizontal circle.**

The tension in the string provides the centripetal force necessary to keep the ball moving in a circular path.

2. **A satellite orbiting the Earth**.

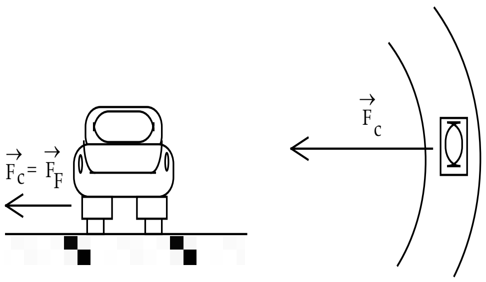


The centripetal force is provided by the gravitational attraction between the earth and the satellite.

This force always acts towards the earth's centre.

3. **Rounding a curved track**

When a motor vehicle rounds a curve the frictional force between the road and the tyres provides the necessary centripetal force.



The maximum centripetal force is equal to the maximum frictional force that can be exerted by the surface on the tyres.

If the frictional force necessary to provide the centripetal force is insufficient, then the car will not follow the required curve and may "run off" the road.

This maximum frictional force depends upon -

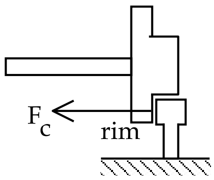
(i) the condition of the surface (e.g. whether dry or not);

(ii) the condition of the tyres.

Since $F_c = \frac{mv^2}{r}$ the faster the car or the tighter the curve, the greater the frictional force needed

4. **Railway tracks**

When a vehicle on a railway track rounds a bend, the track exerts a force on the rim of the wheels. This provides the necessary centripetal force.

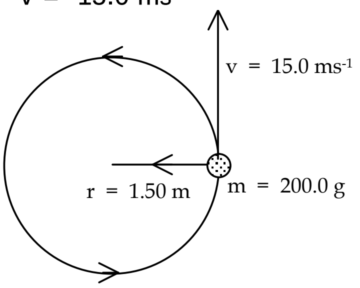


Example 1: A ball of mass 200.0 g is attached to a string of length 1.50 m. The ball is then whirled in a horizontal circle at 15.0 ms^{-1}. Determine the centripetal acceleration and the tension in the string.

m = 200.0 g

r = 1.50 m

v = 15.0 ms^{-1}



(i) $a = \frac{v^2}{r}$

$= \frac{15^2}{1.5}$

$= \underline{1.50 \times 10^2}\ ms^{-2}$

(ii) $F_C = m\,a$

$= 0.2 \times 150$

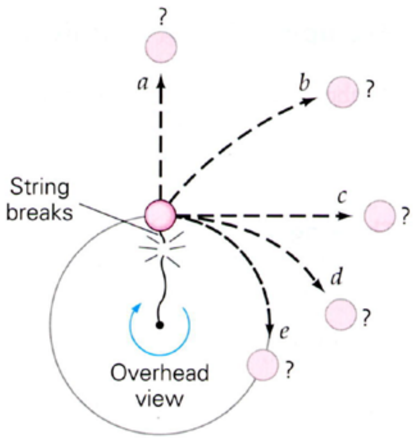
$= \underline{30.0\ N}$

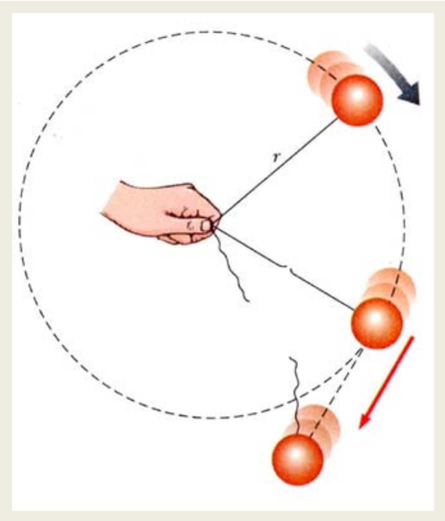
Problem 1: Determine the centripetal acceleration and force exerted on a girl of mass 40.0 kg, if she is to stay on a merry-go-round that completes one rotation every 15.0s, given that she is 5.00 m from the centre of rotation of the merry-go-round.

Centripetal vrs Centrifugal Force

Consider a stone whirled in a horizontal circle via the tension in a string...

In which direction would the stone "fly" if the string was to break?





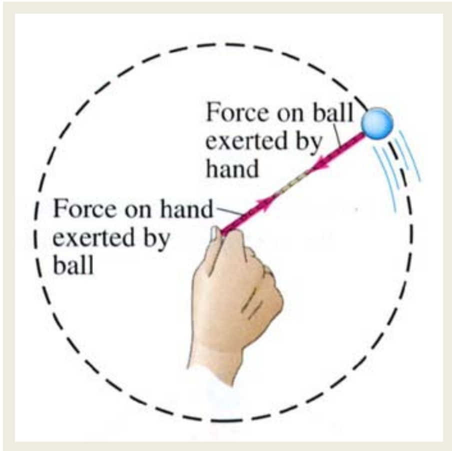
The force is exerted on the stone through the tension in the string.

If this force is removed, the stone travels off in a straight line at its point of release.

This in accordance with Newton's 1st law of motion.

The straight-line path is tangent to the circle at the point of release.

The centripetal force provided by the tension enables the object to maintain a circular path by resisting its inertial (straight-line) tendency.



Centrifugal Force

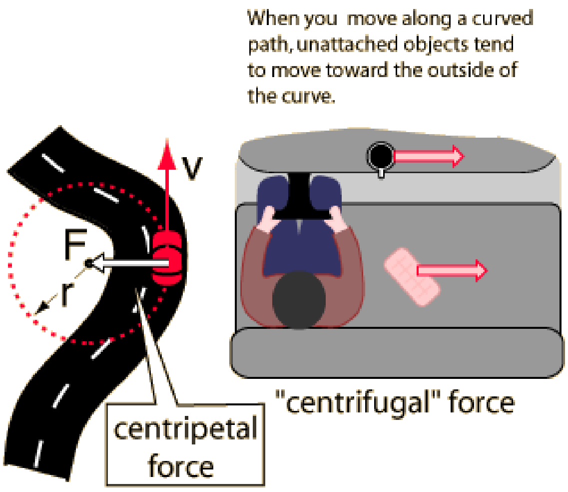
Newton's 3rd law of action/reaction states that for every force on a body there is an equal and opposite force acting on some other body (or bodies).

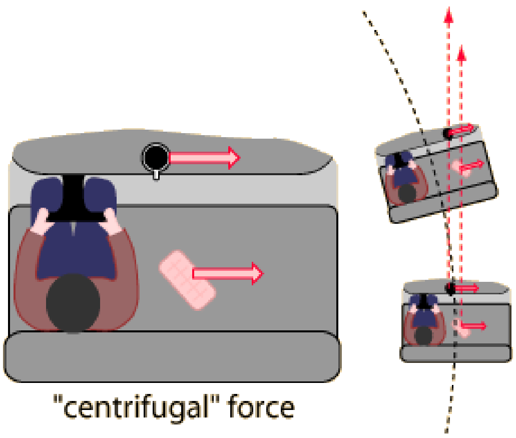
The centripetal force is exerted on the body that is moving in the circular path, while the opposite force is (acting on a second body that is NOT on the body travelling in a circle) is called the centrifugal force.

Centrifugal Forces are used to explain the behaviour of objects from a rotating frame of reference.

A Common Misconception:

Why do the objects seem to slide sideways **away from** the centre of the curved path?





The objects are actually moving in a straight line as their inertia dictates...

Exercise Set 3: Uniform Circular Motion

Data: Radius of Earth = 6.67×10^6 m; Radius of Earth's orbit = 1.50×10^{11} m.

1. Calculate the periods in seconds of a record turntable rotating with rates of:
   a) 33.3 rpm b) 45.0 rpm c) 78.0 rpm (rpm means revolutions per minute)

2. A stone on the end of a string of length 2.0 m completes one circle in 1.5 s.
   What is the tangential speed of the stone?

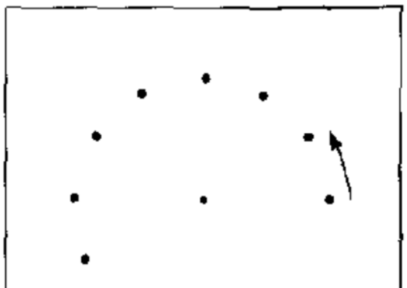
3. The Earth - Moon distance is 3.85×10^5 km. The Moon's period of revolution is 27.3 days.
   a) What is its tangential speed?
   b) How far will it travel in 1 day?
   c) Through what angle will it move in 1 day?

4. A wheel revolving at 6.0 rpm. has a radius of 30 cm.
   a) What is its period of revolution?
   b) Find the tangential velocity of a point on the rim of the wheel.

5. An electron is in a circular orbit about the nucleus of an atom.
   The radius of the orbit is 2.00×10^{-10} m and its speed is 1.00×10^6 ms^{-1}.
   Find the centripetal acceleration of the electron and its period.

6. A space station of radius 50.0 m is in orbit and is spinning at a speed that causes a centripetal acceleration of 10.0ms^{-1} (i.e. Earth gravity conditions). What is the tangential velocity at a point in the outside rim of the station?

7. A multi-flash photograph for a puck undergoing uniform circular motion is shown here. Given a flash rate of 10 flashes per second, determine each of the following for the puck's motion:
   a) tangential velocity b) acceleration c) period of revolution.



Question 7

scale 1mm : 10 mm

8. The Earth undergoes two approximately uniform circular motions. a) What are these motions?
   b) Determine the ratio of the i) periods of motion ii) centripetal acceleration of motion given
   $r_e = 6.37 \times 10^6$m; $r_{earth\ orbit} = 1.50 \times 10^{11}$ m.

9. A 1.00×10^3 kg car begins to skid when travelling 108 kmh^{-1} around a level curve of radius 1.50×10^2 m. Find the centripetal acceleration, the centripetal force and the frictional force between the tyres and the road.

10. A 1.20×10^3 kg car is going around a curve of 100 m radius at a speed of 25.0 ms^{-1}
    Find the centripetal acceleration and the centripetal force.

11. People's responses to high accelerations are checked in a test chamber at one end of a 7.00 m horizontal beam rotated about a vertical axis at its other end. How many revolutions per minute must the beam make to provide an acceleration of 9.00 g's (88.2 ms^{-2})?

12. What is the highest speed an automobile can travel around an 80.0 m radius curve on a level road if the maximum force of friction between the road and the tyres is 0.49 times the weight?

13. Find the centripetal acceleration of an object at the equator due to the rotation of the earth.
    By how much does the rotation of the earth reduce the weight of a 70.0 kg man?

14. How many revolutions per day would the earth have to make for the weight of a body at the equator appear to become zero?

Answers

1. a) 1.80 s b) 1.33 s c) 0.77 s
2. 8.4 ms^{-1} tangential to circle
3. a) 1030ms^{-1} b) 88600 km c) 13.2^0
4. a) 10s b) 0.19 ms^{-1}
5. 5.0 x 10^{21} ms^{-2} towards centre; 119 ns
6. 22.4 ms^{-1}
7. a) 1.0 ms^{-1} b) 5.0 ms^{-2} c) 1.2 s
8. b) i) 1: 365 ii) 5.7: 1
9. 6.00 ms^{-2}; 6.00 x 10^3 N; 6.00 x 10^3 N
10. 6.25 ms^{-2}, 7.50 x 10^3 N
11. 33.9 rev per min. 3.61 ms^{-2}
12. 19.6 ms^{-1}
13. 0.0339 ms^{-2}, 2.37 N
14. 17.1

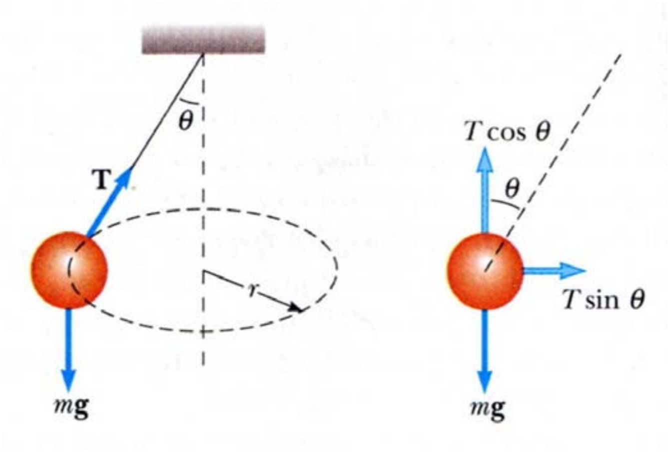
Conical Pendulum

The bob on a pendulum can be made to describe a circular path as shown in the diagram.

The conical pendulum shown consists of a cord connected to a bob.

The tension in the cord provides a force T that can be resolved into two components.

The vertical component of the tension in cord (Tacos θ) provides an upward force which balances the weight of the bob (mg).



$T \times \cos\theta$ = mg ... (1)

The horizontal component ($T \times \sin\theta$) is not balanced and provides the necessary unbalanced centripetal force required to enable the bob to follow a circular path.

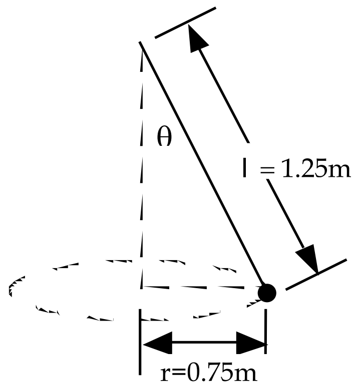
$T \times \sin\theta = \frac{mv^2}{r}$ ... (2)

(2) ÷ (1) $$\frac{T \sin\theta}{T \cos\theta} = \frac{mv^2}{r\,mg}$$

$$\therefore \quad \tan\theta = \frac{v^2}{rg}$$

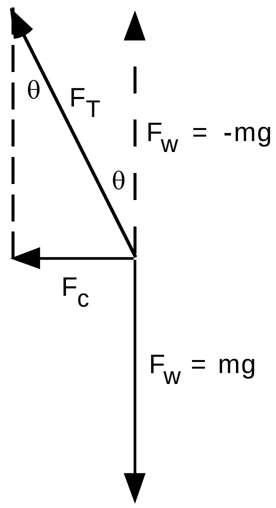
Example 1: A body of mass 0.50 kg is hung from a string 1.25 m long and swung around in a horizontal circle of radius 75 cm. Find:
a) the tension in the string and
b) the period of revolution of this 'conical pendulum'.

$$\sin\theta = \frac{r}{l} = \frac{0.75}{1.25} = 0.6; \quad \theta = \mathbf{\underline{36.9}°}$$



a) $$\frac{F_W}{F_T} = \frac{mg}{F_T} = \cos\theta$$

$$\therefore F_T = \frac{mg}{\cos\theta} = \frac{0.50 \times 9.80}{\cos 36.9} = \mathbf{6.12\ N}$$



b) $$F_C = F_T \sin\theta = \frac{mv^2}{r}; \text{ but } v = \frac{2\pi r}{T}$$

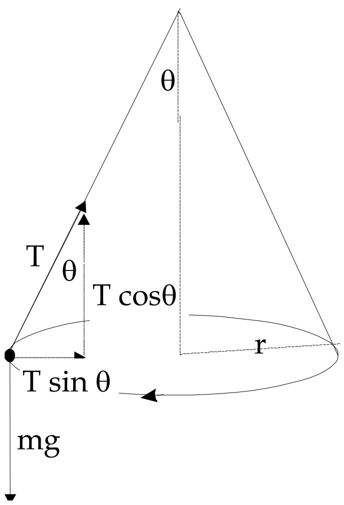
$$\therefore F_T \sin\theta = \frac{m(2\pi r)^2}{rT^2};$$

$$\therefore T^2 = \frac{m 4\pi^2 r}{F_T \sin\theta}$$

$$T^2 = \frac{0.50 \times 4 \times \pi^2 \times 0.75}{6.12 \times 0.60};$$

T = 2.00 s

Example 2: A conical pendulum consists of a string connected to a 0.750 kg bob. The bob is observed to move in a circle of radius 0.500 m with a steady speed of 1.50 ms^{-1} Determine the angle of the string to the vertical and the tension in the string.



$$T\sin\theta = \frac{mv^2}{r} \ ....(1)$$

$$T\cos\theta = mg \ ....(2)$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2}{mgr} \ ....(1) \div (2)$$

$$\tan\theta = \frac{v^2}{gr} = \frac{(1.5)^2}{9.8 \times 0.5} = 0.45$$

$$\theta = 24.6°$$

$$T\cos\theta = mg$$

$$\therefore T = \frac{mg}{\cos\theta} = \frac{0.75 \ x \ 9.8}{\cos 24.6}$$

$$= 8.08N$$

Problem 1 Using the same pendulum in the example above, determine the velocity of the bob if the angle that the string makes with the vertical is reduced to 15.0°.

Moving around Bends

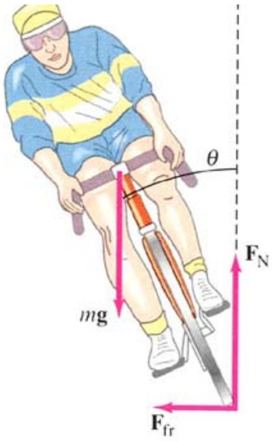
Consider each of the following: Why is it necessary to “lean into” a bend ?

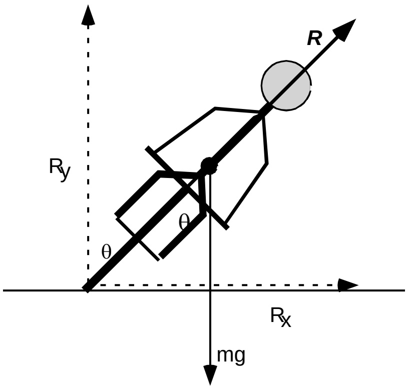


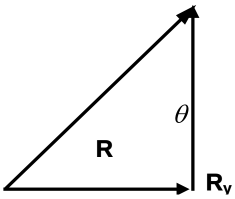
Banking: Bicycles and Motor Bikes

To enable a bicycle (or motor bike) to round a bend, the bicycle must lean at an angle to the vertical.

The tighter the curve or greater the velocity, the greater the angle (θ) to the vertical.







The reaction force **R** exerted on the tyres of the bicycle must pass through the centre of gravity of the bike and rider for stability.

The horizontal component of R is $\mathbf{R_x}$.

This provides the necessary centripetal force required for circular motion.

This component is provided by friction.

If there is insufficient friction between the ground and the tyres, the motorcycle will skid.

The vertical component of R is $\mathbf{R_y}$ balances the weight of the rider and cycle.

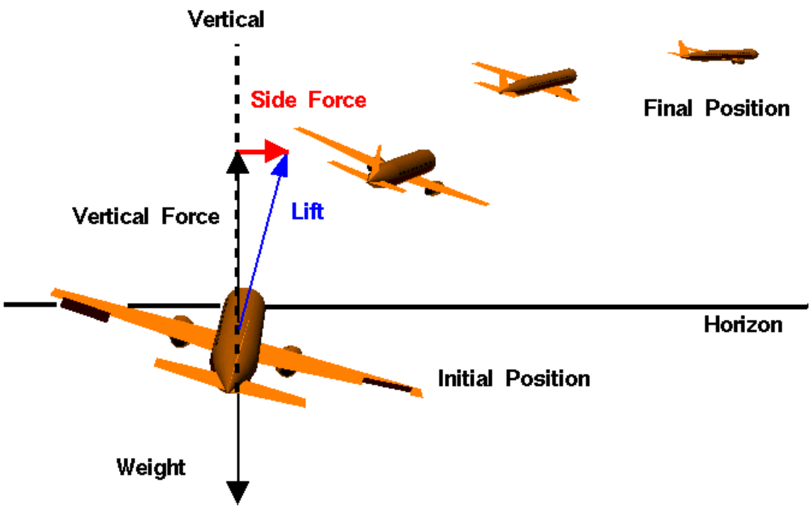
$$R_x = R \times \sin\theta = \frac{mv^2}{r} \quad \text{and} \quad R_y = R \times \cos\theta = mg$$

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$

Note that this last equation is identical to the final equation for the conical pendulum.

Aircraft Turns



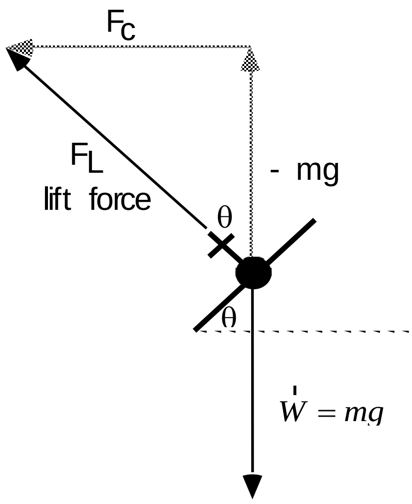
An aircraft is turned by 'banking' - the aircraft rolls slightly about its longitudinal axis.

As a result the lift force provides a vertical component to balance the weight of the aircraft and a horizontal component which provides the centripetal force.

If an aircraft banks but the pilot makes no further adjustments it will lose height. Why ?

$$\tan\theta = \frac{F_c}{mg} = \frac{mv^2}{rmg}$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg}$$



Thus the tighter the curve, or the greater the velocity, the greater the banking angle.

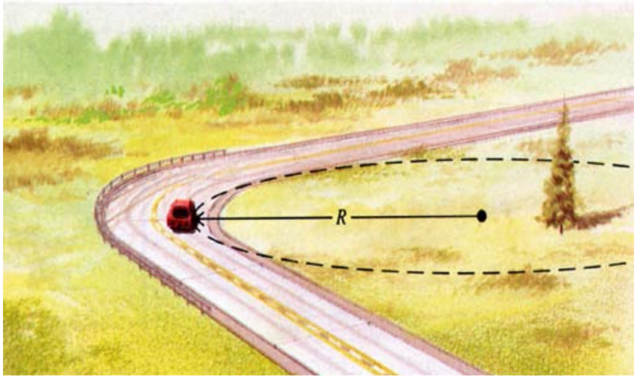
Banked Circular Tracks

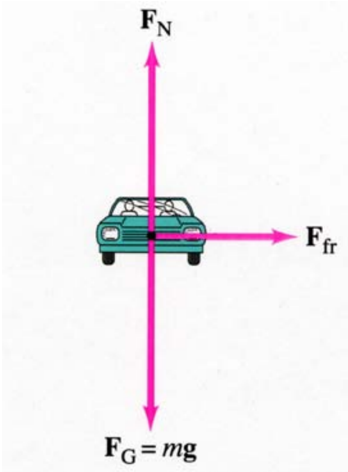
It is common to provide banking on the bends of velodromes and racing car circuits to allow safer (and faster) cornering at speed.

Road Banking

Similar but less pronounced examples of this are often seen on country roads in Western Australia.

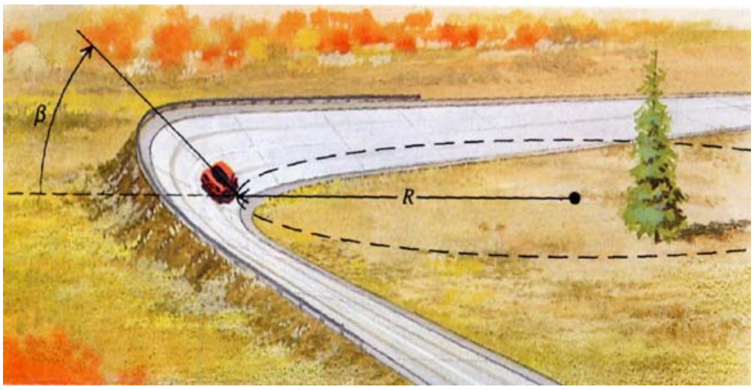
All of the centripetal force required to take a bend on the flat road must be provided by the tyres (via friction).



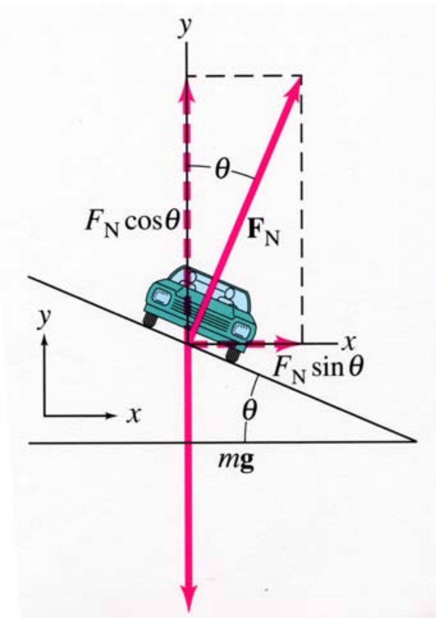


Roads can be banked so that a component of the reaction force provides the centripetal force to keep the car going around, without any help from friction. i.e. the track could be frictionless.

In constructing safer high speed roads, engineers bank the road on the bends so that the outside is higher than the inside.



The banked slope helps to resists the inertial tendency by providing some of the required centripetal force and thereby relying less on tyres and road conditions.



The angle of banking is θ. And $\theta \, \alpha \, v^2$ and $\frac{1}{r}$,

and is independent of mass (from $\tan\theta = \frac{v^2}{rg}$)

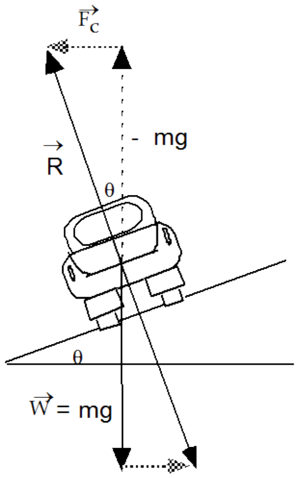
A particular banked track is ideal for only one velocity.

From F_c = mg tanθ $\qquad \tan\theta = \frac{F_c}{mg} = \frac{mv^2}{rmg}$

$\tan\theta = \frac{v^2}{rg}$ $\qquad$ also $\quad \mathbf{\tan\theta = \frac{a_c}{g}}$

also **F_c = mg × tanθ**

Example 1: At what angle must a road be banked such that a vehicle travelling at 90.0 kmh^{-1} may round curve of radius 150.0 m, without any frictional force being applied ?



$$F_c = \frac{mv^2}{r} \quad \text{but} \quad \tan\theta = \frac{F_c}{mg}$$

$$\therefore \tan\theta = \frac{mv^2}{mgr} = \frac{v^2}{rg}$$

$$\tan\theta = \frac{25^2}{9.8 \times 150} = 0.425$$

i.e. $\underline{\theta = 23^o}$

Problem 2 A speedway driver travelling down the straight at 1.10 x 10^2 kmh^{-1} throws his car into a turn banked at an angle of 10.0º to the horizontal.

a) What is the turning radius assuming friction does not contribute to centripetal force?

b) What is the value of the centripetal acceleration in g's?

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Problem 3: A racing track incorporates a bend with a radius of 2.00×10^2 m. If the bend is to be banked such that a car moving at 1.50×10^2 kmh^{-1} can negotiate it without any frictional aid, determine the angle of the bank.

Problem 4: During a test flight, a jet fighter flying at 2.50×10^2 ms^{-1} is banked at an angle of 20.0° to the horizontal. Determine i) the radius of the turn ii) the number of 'g's exerted on the pilot.

EXERCISE 4: Banked Tracks

1. A curve of 220 m radius is banked at 18.0^0. At what speed must an automobile go around this curve if no frictional forces are to be used to keep the automobile on its circular path?

2. What is the angle at which a speedway must be banked for cars running at 50.0 ms^{-1} if the radius of curvature is 350 m and no frictional force is involved?

3. A 0.60 kg pendulum bob hangs from the roof of a moving van. If the van is travelling 35.0 ms^{-1} around a curve of 245 m radius, find the angle which the cord makes with the vertical when the bob is at rest relative to the van. What is the tension in the cord?

4. A cord 1.50m long supports a 2.00 kg pendulum bob which traces out a conical pendulum. The angle the pendulum bob makes with the vertical, θ is 30.0^o.
   a) What is the speed of the pendulum bob?
   b) Find the tension in the cord.

5. You are in a playground riding on a conical swing which causes the 2.40 m chain that is supporting you to swing out so its hand-ring is 1.50 m from the central pole. Assume your centre of mass is 1.60 m below your outstretched hands. How fast will you 'fly off' if you let go? i.e. What would be the speed of your centre of mass?

6. A fighter pilot travelling horizontally at 1.40 $x10^3$ kmh^{-1} throws his aircraft into a turn by banking his aircraft at an angle of 60.0^o to the horizontal.
   a) What is the radius of his turn?
   b) What is the value in $g^{'s}$ of the centripetal acceleration?

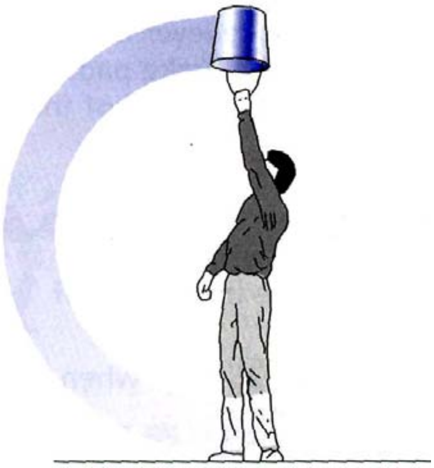
Answers:

1. 26.5 ms^{-1}
2. 36.1^0
3. 27.0^0, 6.60 N
4. 2.06 ms^{-1}, 22.6 N
5. 4.43 ms^{-1}
6. a) 8.91 km b) 1.73

Vertical Circular Motion

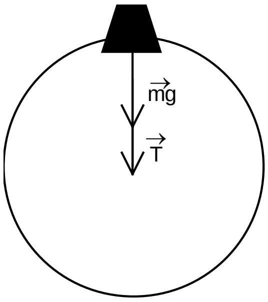
A body moving through a vertical circle will not "fall" out of the loop so long as the centripetal force required to maintain the circular motion exceeds the weight of the body.

Consider a bucket attached to a rope being whirled in a vertical circle.



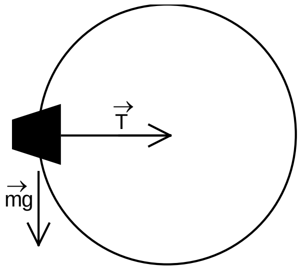
At the top of the circle the centripetal force required to maintain circular motion is given by:

$$F_c = T + mg$$



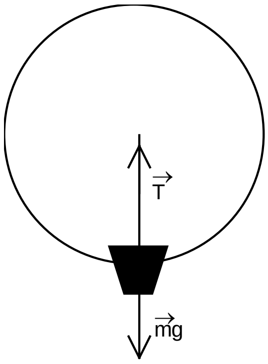
At the side of the circle the centripetal force required to maintain circular motion is given by:

$$F_c = T$$

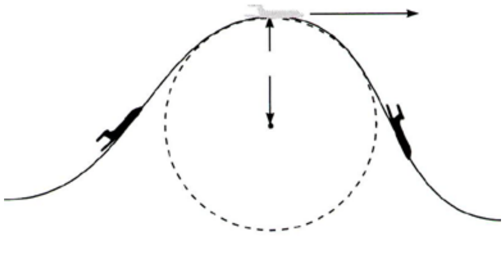


At the bottom of the circle the centripetal force required to maintain circular motion is given by:

$$F_c = T - mg$$



The above reasoning can be applied to any body moving in a vertical circle.

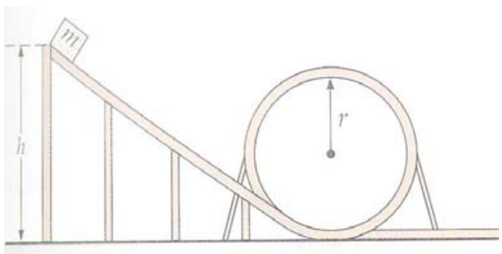


- For an aeroplane describing a vertical loop the force "T" represents the “Lift”.

  This is the wing loading of the plane
  - the force the wings apply to the plane.

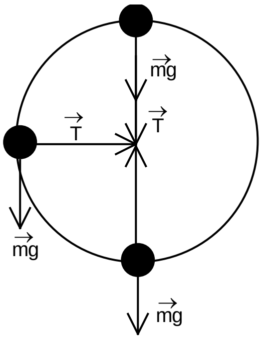
- For a roller coaster car negotiating a looped track the force "T" is the force of the track on the car.

  Note that the reaction force of the car on the track will be equal but opposite.



Whatever maintains the body in a circular path (i.e. string, rails, thrust etc) must provide sufficient force such that the resultant force acting on the body is the required centripetal force.

Example 1: A stone of mass 50.0 g is whirled at 4.00 ms^{-1} in a vertical circle by a string 0.500 m long. Find the tension in the string at the:
a) top of the circle.
b) bottom of the circle.
c) side of the circle.



(a) at the top

$F_c = T + mg$

□ $T = F_c - mg = \frac{mv^2}{r} - mg$

□□□□□□ $T = m\left(\frac{v^2}{r} - g\right)$

$= 0.05 \times \left(\frac{4^2}{0.5} - 9.8\right)$

$T =$ **1.10N**

(b) at the sides

$F_c = T = \frac{mv^2}{r} = \frac{0.05 \times 4^2}{0.5}$

$F_c =$ **1.60 N**

(c) at the bottom

$F_c = T - mg$

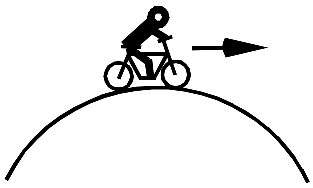
$T = F_c + mg$

$= \frac{mv^2}{r} + mg$

$T = 1.6 + 0.05 \times 9.8$

$=$ **2.09 N**

Problem 1: A man of mass 80.0 kg on a motor cross cycle rides over the crest of a hill that has a radius of curvature of 50.0m. Determine the reaction force on the rider at the top of the crest if he rides his bike at 15.0 ms^{-1}.

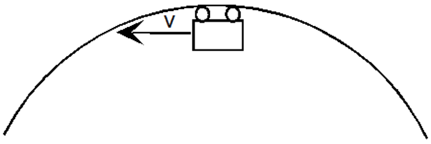


Minimum Velocity for Vertical Circle

The minimum velocity necessary for a body to achieve a vertical circle occurs when:

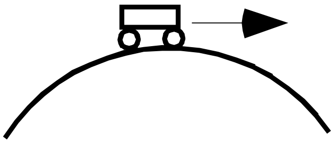
$$F_c = mg \qquad \frac{mv^2}{r} = mg \qquad \text{i.e. } \frac{v^2}{r} = g$$

Thus $v = \sqrt{rg}$ this is termed the __critical velocity__.



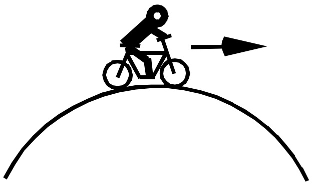
For a body following a path inside the circle v is the minimum velocity required so that the body will not fall out of its path.

If $v < \sqrt{rg}$ then the body will "fall" from its vertical circular path.



Note if a body moves over the top of a curve of radius r then the maximum speed that the body can have without becoming "airborne" is given by $v_{max} = \sqrt{rg}$

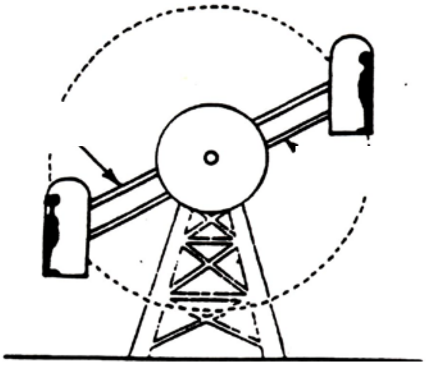
Problem 2: Determine the maximum speed that the bike, in the problem on the previous page, can travel at without the bike becoming airborne.



Problem 3: A Ferris wheel has a radius of 4.00 m.

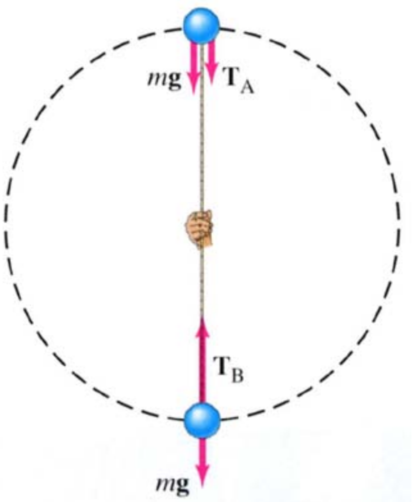
a) What is the maximum period of rotation of the wheel if the riders are to experience no more than a 20.0% reduction in weight?

b) What period of rotation would be required if the riders were to experience weightlessness?



Non –Uniform Circular Motion

The centripetal force and therefore the speed of an object in a vertical circular motion is not uniform.



The reaction (tension) varies throughout. Top: minimum
Bottom: maximum

The speed varies throughout. Top: minimum
Bottom: maximum

The centripetal force varies throughout. Top: minimum
Bottom: maximum

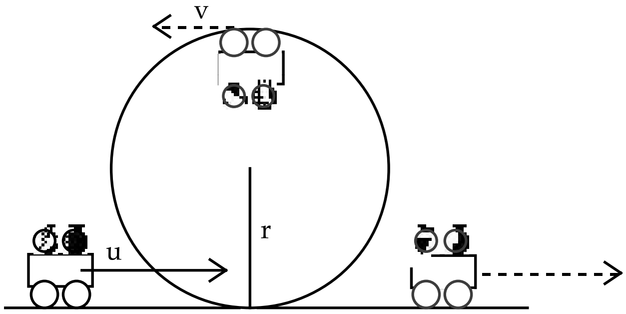
Top: minimum $F_C = F_{NET} = F_R + F_W$

Bottom: maximum $F_C = F_{NET} = F_R - F_W$

Energy Considerations

When a body moves through a vertical loop there is an interchange of kinetic and potential energies. As a consequence, both the centripetal acceleration and the centripetal force will vary with position.

Consider the following. At a fun park a roller coaster car is required to "loop the loop". Determine the minimum velocity necessary.



Energy is conserved

$$E_{K(i)} = E_{K(f)} + \Delta E_P$$

$$\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mgh$$

and $h = 2r$

$$u^2 = v^2 + 4gr \quad ... \quad (1)$$

Critical velocity at top of loop.

$$\frac{v^2}{r} = g$$

$$v^2 = rg \quad ... \quad (2)$$

Substitute (1) in (2)

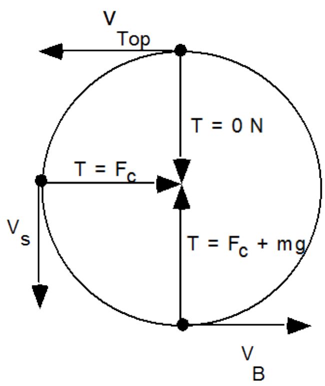
$$u^2 = rg + 4gr$$

$$u^2 = 5rg$$

$$\mathbf{u = \sqrt{5rg}}$$

Problem 4: At a fun park a roller coaster is required to negotiate a vertical loop of radius 10.0 m. What is the velocity that the roller coaster car can enter the loop at A if the occupants are to experience a “g” force of 0.200 g at the top of the loop?

Example 2: A mass of 0.500 kg is swung in a vertical circle on a string of length 1.50 m. The mass is travelling at the minimum velocity needed to maintain circular motion. Calculate the tension in the string at the a) top b) side c) bottom of the circle



If the mass is travelling at v_{min} at the top,

then T_{top} = 0 N and $F_c = mg$

i.e. $\frac{mv^2}{r} = mg$; $\frac{v^2}{r} = g$; $v = (gr)^{\frac{1}{2}}$

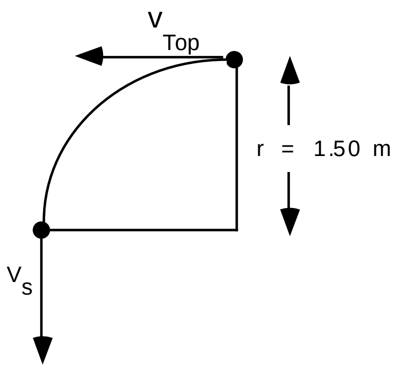
$v = (9.80 \times 1.50)^{\frac{1}{2}}$

v = <u>3.83 ms^{-1}</u>

a) T_{Top} = 0 N

b) <u>At the side</u> i.e. half way down, $F_c = T$;

i.e. $T = \frac{mv^2}{r}$, however v has increased due to effect of gravity on the mass.



$E_{k\ side} = E_{k\ top} + \Delta E_p$

$\frac{1}{2}mv^2_{side} = \frac{1}{2}mv^2_{top} + mgr$

$v_{side}^2 = v_{top}^2 + 2gr$

$= (3.83)^2 + 2 \times 9.80 \times 1.50$

i.e. v_{side} = <u>6.64 ms^{-1}</u>

$\therefore T_{side} = \frac{mv^2_{side}}{r} = \frac{0.500 \times (6.64)^2}{1.50}$ = <u>14.7 N</u>

c) <u>At the bottom</u>

$E_{k\ bottom} = E_{k\ top} + \Delta E_p$

$\frac{1}{2}mv^2_{bottom} = \frac{1}{2}mv^2_{top} + mg2r$

$v^2_{bottom} = v^2_{top} + 4gr = (3.83)^2 + 4 \times 9.80 \times 1.50$

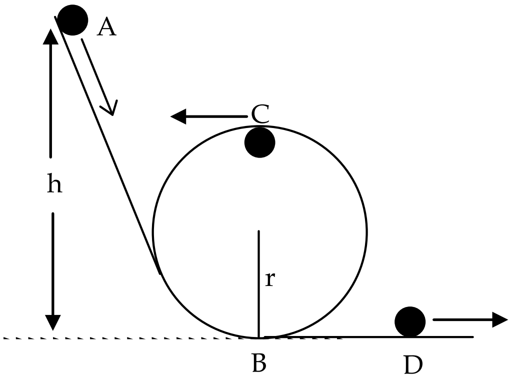
i.e. v_{bottom} = <u>8.57 ms^{-1}</u>

□ $T_{bottom} = F_c + mg = \frac{mv^2_{bottom}}{r} + mg$

$= \frac{0.500 \times (8.57)^2}{1.50} + 0.500 \times 9.80$

i.e. T_{bottom} = <u>29.4 N</u>

Example 3: Determine the minimum height a body can fall so that it can just vertically loop a circle of radius r.



Consider a body allowed to fall freely from height "h", above the base of a loop of radius r.

The velocity at C is determined by assuming the kinetic energy of the body at C is equal to the change in potential energy between A and C.

i.e. $E_K = \Delta E_P$

$\Rightarrow \frac{1}{2} mv^2 = mg(h - 2r)$

So, $v^2 = 2g(h - 2r)$ ... (1)

For critical velocity at C $F_c = mg$ $\frac{mv^2}{r} = mg$ or $v^2 = rg$ ... (2)

Equate v^2 from (1) and (2), giving $2g(h - 2r) = rg$

So, $h = \frac{5r}{2}$

EXERCISE 5: Vertical Circles

1. A stunt pilot flies a plane at a **constant speed** of 50.0 ms^{-1} through a vertical circle of radius 50.0 m. a) What is the centripetal acceleration acting during this flight?
   b) What are the maximum and minimum accelerations on the pilot caused by the flight?

2. What is the smallest speed an aeroplane can have making a vertical loop with a radius of 720 m if objects in the plane are not to start dropping at the top of the loop?

3. A 0.45 kg mass is suspended from a cord 1.50 m long to form a simple pendulum.
   If the mass is pulled to one side until it is raised 100 mm and then released, find:
   a) the velocity of the mass, and
   b) the tension in the cord at the bottom of the swing.

4. A 8.00 x 10^3 kg aircraft with a constant speed of 540 kmh^{-1} pulls out of a vertical dive along an arc of a circle of radius 500 m. Find: a) the centripetal acceleration, and
   b) the total lift required at the bottom of the dive.

5. A 0.20 kg ball is whirled in a vertical circle on the end of a string 0.60 m long.
   a) At what speed will the tension in the string be zero at the top of the circle?
   b) What will the tension at the bottom be if the tension drops to zero at the very top?
   (**Hint:** the ball gains speed as it descends).

6. A small cart on the circular loop-the-loop track of the figure below can be approximated as a mass 'm' sliding on a frictionless track.

a) Find the smallest height 'h' in terms of the radius 'r' from which the mass can be released and still remain in contact with the track throughout the path.

b) If the cart starts from a height of 3.00 r, what is its acceleration at the top of the loop?

c) With what force does the track press down on the cart at the top?

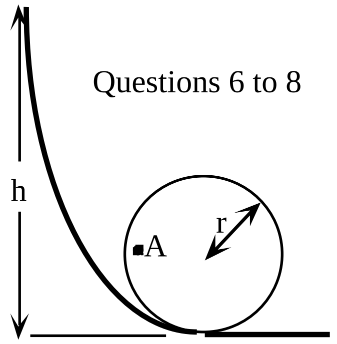
d) When the cart starts at h = 3.00 r, find the horizontal and vertical components of its acceleration at point A on the end of a horizontal diameter.

7. If the cart in the previous question starts from a height of 5.00 r, find its kinetic energy and the force it exerts on the track at each of the following:

a) At the bottom.

b) At the top of the circle.

c) At point A.



8. Assume the track in the figure above has a radius of 0.30 m and is frictionless.
Find the speed and the force exerted on the track for a 40.0 g cube that is released from a height of 0.90 m at the following positions: a) At the bottom. b) At the top. c) At point A.

Answers

1. a) 50 ms^{-2} b) max 60 ms^{-2}, min 40 ms^{-2}
2. 84.0 ms^{-1}
3. a) 1.40 ms^{-1}; b) 5.00 N
4. 4.38×10^5 N
5. a) 2.42 ms^{-1}; b) 11.8 N
6. a) 2.50 r; b) 2.00 g, mg, 4.00 g, g
7. a) 5.00 mgr, 11.0 mg;
   b) 3.00 mgr 5.00 mg;
   c) 4.00 mgr, 8.00 mg
8. a) 2.74 N; b) 0.392 N; c) 1.57 N

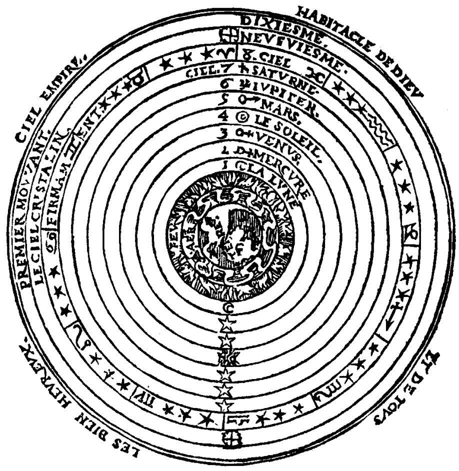
Universal Gravitation

The Law of Gravitation

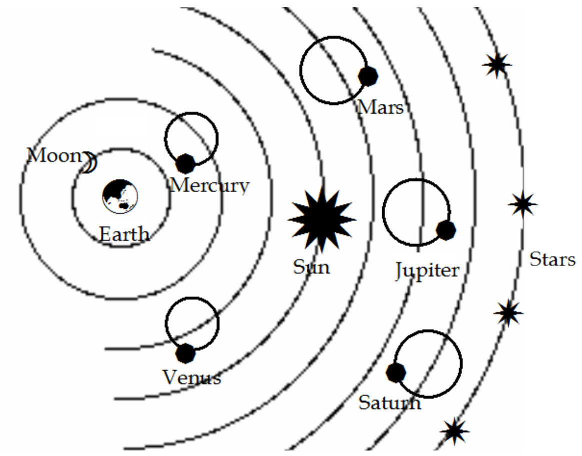
An Historical Perspective

The time will come when diligent research over long periods will bring to light things which now lie hidden. A single lifetime, even though entirely devoted to the sky, would not be enough for the investigation of so vast a subject . . . And so this knowledge will be unfolded only through long successive ages. There will come a time when our descendants will be amazed that we did not know things that are so plain to them . . . Many discoveries are reserved for ages still to come, when memory of us will have been effaced. Our universe is a sorry little affair unless it has in it something for every age to investigate . . . Nature does not reveal her mysteries once and for all.

—Seneca, Natural Questions, Book 7, first century



For thousands of years, man has attempted to make sense of the motion of "heavenly bodies".



Ptolemaic System with the Earth at the Centre of the Universe

In Greek mythology, the sun was depicted as the god Apollo driving his fiery chariot across the sky. The Greek philosophers Pythagoras and Aristotle, amongst others, believed the earth to be the centre of the universe.

This "geocentric" theory involved the sun, moon, stars and the planets revolving about the Earth.

As early as 260 BC, Aristarchus had predicted a "heliocentric" model in which the Earth and the planets revolved about the sun. This theory was largely disregarded and the geocentric model persisted.

Around 200 AD, Ptolemy proposed a model of the universe that had the stars on a large transparent dome moving around the earth at a constant rate.

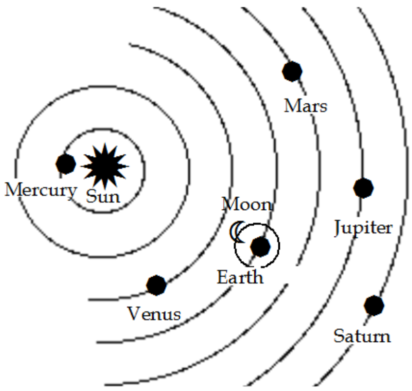
According to the theory the sun and the moon were situated on smaller concentric domes and revolved about the earth's centre at different rates.

The motions of the planets[1], however, were observed to move generally in the same direction as the stars, but periodically retrogressed (i.e. appeared to move backwards).

Ptolemy explained this by assuming they moved in small circular paths about a centre that moved in a larger circular path associated with the transparent dome.

For years Nicholas Copernicus had studied the motion of the planets and in 1543 was able to show that if a heliocentric model for the solar system was adopted, then the movement of the planets could be described in terms of simple orbits about the sun.

[1]The largest planets in the solar system (Jupiter, Saturn, Uranus and Neptune) are known as the Jovian planets. Mercury, Venus, Earth and Mars are known as the Terrestrial planets.



Nicolaus Copernicus (1473–1543).

Copernican System with the Sun at the Centre of the Solar System

Kepler's Laws

Johan Kepler analysed the data collected by Tycho Brahe. Brahe had meticulously observed and measured the apparent motion of the stars and the planets over many years.

Kepler used the results of his analysis to enunciate three laws to accurately describe the motion of the planets viz:

1) Law of Ellipses
   The orbit of each planet is an ellipse and the sun is at one of the foci;

2) Law of Equal Areas
   The speed of the planets varies in such a way that a line joining the planet and the sun will sweep out equal areas in equal times;

3) Law of Harmony
   The square of the period of revolution of elliptical orbits is proportional to the cube of the 'radius'.
   For essentially circular orbits this can be represented by: $\mathbf{T^2 = kr^3}$ (k is a constant)

Some Mathematics

Consider the circular motion of the earth about the sun.

The centripetal force is given by:

$$F_c = \frac{mv^2}{r} \quad \ldots \quad (1)$$

The velocity of the earth is given by:

$$v = \frac{2\pi r}{T} \ldots \quad (2)$$

Substitute equation (2) into equation (1)

$$F_C = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{m 4\pi^2 r^2}{rT^2} = \frac{4\pi^2 m r}{k r^3}$$ (since $T^2 = kr^3$ from Kepler's 3rd Law)

Thus $F_c = \dfrac{4\pi^2 m}{k r^2} \ldots$ **(3)** where m is the mass of the earth.

Newton correctly concluded that the centripetal force was provided by the Earth's gravitational attraction for the sun.

This force depends not only on the mass of the Earth but also the mass of the sun.

Therefore the value of the constant "k" in equation (3) must incorporate the mass of the sun (M).

i.e. $\frac{4\pi^2}{k} = GM$ where G is a universal constant.

Thus the gravitational attraction between the sun and the Earth is given by: $F = G\frac{mM}{r^2}$

A general statement of Newton's Law of Gravitation

Between two bodies there exists a force of attraction which is directly proportional to the product of their masses and inversely proportional to the square of their distance apart.

$$F = G\frac{m_1 m_2}{d^2}$$

G is termed the universal gravitational constant.

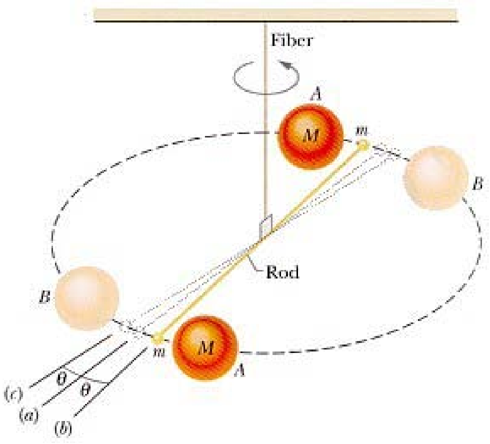
$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

- This law holds for any two bodies - not just planets.
- 'G' holds the same value throughout the universe.
- 'F' is a mutual force - i.e. each of the bodies will attract the other body with an equal Force.

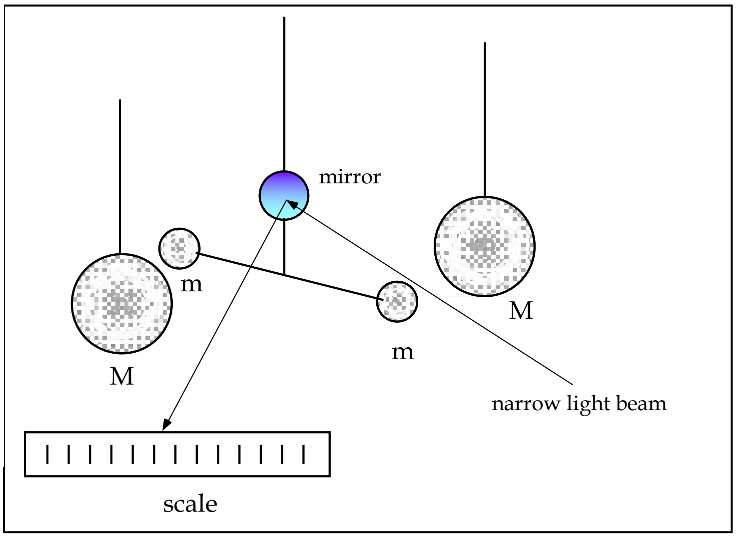
The Universal Gravitational Constant

Theory does not predict the value of G, thus it is determined empirically (i.e. experimentally).

The value of G was first measured by Henry Cavendish in 1798 using a "Cavendish" balance.

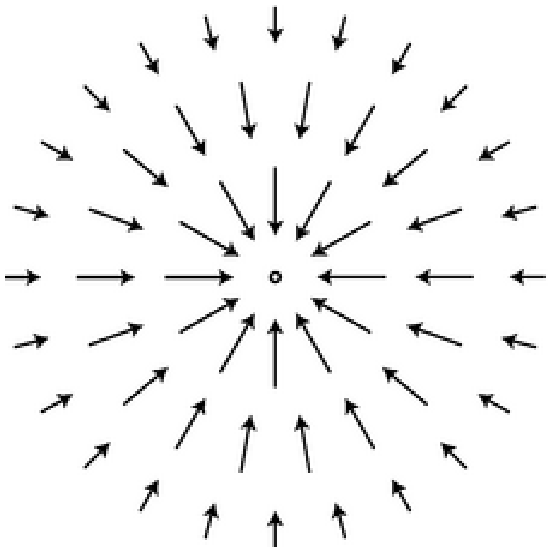


Cavendish Balance



The gravitational attraction between the masses m and M produces a torque which can be measured by reflecting a light beam from the mirror.

Gravitational Fields



All celestial and terrestrial bodies have associated gravitational fields.

A gravitational field is a region where a body experiences a force due to the mass of another body.

The Earth has a gravitational field associated with its mass.

A body located in the Earth's gravitational field will experience a force attracting it to the Earth's centre.

The strength of a gravitational field at a given point is c mass.

Thus gravitational field strength (g) is given by $g = \frac{F}{m}$ or $F = mg$

- The **<u>SI unit</u>** for gravitational field strength is Nkg^{-1}
- The force **F** is actually the weight of the test mass, and it acts in the same direction as **g**.

Acceleration Due To Gravity

Consider a body of mass "m" placed in the earth's gravitational field.

The force on the body $F = mg = \frac{GmM_e}{r^2}$

where M_e is the mass of the earth
and r is the distance to the earth's centre.

$$\boxed{\mathbf{g = \frac{GM_e}{r^2}}}$$

<u>Example 1:</u> The moon has a mass of 7.34×10^{22} kg and is maintained in an orbit of radius 3.84×10^5 km about the Earth by its gravitational force of attraction. Determine the gravitational force between the Moon and the Earth given that the Earth's mass is 5.98×10^{24} kg.

$m_1 = 7.34 \times 10^{22}$ m

$m_2 = 5.98 \times 10^{24}$ kg

$r = 3.84 \times 10^8$m

$$\mathbf{F} = G\frac{m_1 m_2}{d^2}$$

$$\mathbf{F} = (6.67 \times 10^{-11}) \times \frac{7.34 \times 10^{22} \times 5.98 \times 10^{24}}{(3.84 \times 10^8)^2}$$

$$\mathbf{F} = 1.99 \times 10^{20} \text{ N}$$

Example 2: What is the value of 'g' at the surface of a planet whose mass is three times that of Earth and whose radius is twice that of Earth?

$g_e = 9.80\ \text{Nkg}^{-1}$

$$g_p = \frac{Gm_p}{r_p^2} = \frac{G(3M_e)}{(2r_e)^2} = \frac{3GM_e}{4r_e^2}$$

$m_p = 3M_e$

i.e. $$g_p = \frac{3g_e}{4} = \frac{3 \times 9.8}{4}$$

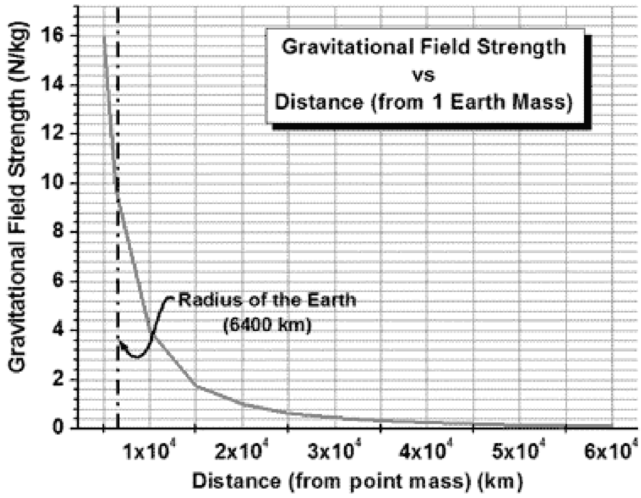
$r_p = 2R_e$

$g_p = ?$

g_p = 7.35 Nkg^{-1}

Variation of "g" with height.

The acceleration due to gravity is inversely proportional to the square of the distance to the Earth's centre.



Consider a body at height "h" above the Earth's surface:

$$\frac{g}{g_s} = \frac{r_e^2}{(r_e + h)^2}$$

(g_s is the acceleration due to gravity at the Earth's surface)

Variation of "g" with depth.

It can be shown that "g" is directly proportional to the distance from the centre of the Earth. Thus g decreases linearly with depth. There is zero gravity at the Earth's centre.

Variation of "g" with latitude.

g varies from place to place on the Earth's surface.

This variation is due to: (i) the equatorial radius being greater than the polar radius;

(ii) the effect of the Earth's rotation.

(iii) the varying densities of the rocks beneath the observer.

At the equator part of the Earth's gravitational force is used to provide the necessary centripetal force.

Thus the apparent pull of the Earth (as measured) is less by about $3.4 \times 10^{-2}\ \text{ms}^{-2}$

The non spherical shape of the Earth accounts for a difference of about $1.8 \times 10^{-2}\ \text{ms}^{-2}$.

The difference between the value measured at the equator and the poles = $5.2 \times 10^{-2}\ \text{ms}^{-2}$.

EXERCISE 6: Newton's Law of Universal Gravitation

Data: Universal Gravitational Constant G = 6.67 x 10^{-11} Nm^2 kg^{-2}
The mass of the Earth is 6.0 x 10^{24} kg
The mass of the Moon is 7.4 x 10^{22} kg
The distance between the Earth and the Moon is 3.84 x 10^{8} m.

1. What is the force of attraction between two objects of mass 100 kg placed 50 cm apart?

2. Find the force of gravitational attraction between:
   a) the Earth and the Moon
   b) a proton (m_p =1.67x10^{-27} kg) and an electron (m_e =9.11x10^{-31} kg) separated by 10^{-10} m .

3. A scientist performs an accurate 'Cavendish type' experiment to determine the value of the universal gravitational constant G. The force between two known masses, separated by a known distance was found. If a force of 1.68 x 10^{-5} N attraction was found after placing two cylinders of mass 10.00 kg apart by a distance of 2.00 cm, what value does this generate for G?

4. Calculate the force of attraction between two super tankers, each of mass 1.00 x 10^{5} t, separated by 2.0 m in a harbour. (Each tanker has a width of 18 m.)

5. A satellite orbits the Earth at height of 600 km.
   What is its mass if it is attracted to the Earth by a force of 719 N? (R_E = 6.4 x 10^{6} m.)

6. At what height above the Earth's surface must a satellite of mass 200 kg be, so that the force of attraction to the Earth is 1200 N?

7. The gravitational force between two objects X and Y is F when separated by a distance r.
   In terms of F, calculate the new force if:
   a) the distance between X and Y is doubled.
   b) the distance between X and Y is bought to one third of the original distance (r) and the mass of X is quadrupled.
   c) the masses of X and Y both double and the distance separating them triples.

8. Given that the Earth has a mass of 6.0 x 10^{24} kg and a radius of 6400 km, find the acceleration due to gravity on its surface.

9. Calculate the acceleration due to gravity at a point:
   a) 100 km from the Earth's surface
   b) at a distance the same as the Moon's orbit (3.84 x 10^{8} m).

10. What would be the acceleration due to gravity on a planet with twice the Earth's mass and one quarter its diameter?

11. The acceleration due to gravity on a planet is 6.25 ms^{-2}.
    What is the planet's radius if its mass is 5.4 x 10^{23} kg?

12. What is the weight of an 80 kg mass on a planet whose mass is 7.2 x 10^{24} kg and radius 8.0 x 10^{6} m?

13. What would be the weight of a 50 kg person on the Sun given that the Sun has a mass of 2.0 x 10^{30} kg and a radius of 700 000 km.

14. Two satellites orbit the Earth.
    Satellite A has a mass of 1.00 x 10^{4} kg and is in orbit at an altitude of 600 km.

    Satellite B has a mass of 4.00 x 10^{4} kg and is in orbit at an altitude of 3600 km.
    Which experiences the greater force of gravitational attraction to the Earth?

15. A space ship is caught between two black holes A and B. It is 20.0 million km from black hole A (mass 1.00×10^{31} kg) and 5.00 million km from black hole B (mass 5.00×10^{30} kg). To which black hole will the ship move?

16. A test mass travels in a straight line toward the Moon from the Earth. At what distance from the Moon will the gravitational attraction of the Earth and the Moon cancel?
(Treat the Moon and the Earth as point masses)

17 Calculate the value of the gravitational constant G, if the radius of the Earth is 6.4×10^{6} m and its average density is 5.5×10^{3} kgm^{-3}.

18 A certain planet has 1.5 times the density of the Earth yet its acceleration due to gravity is the same as the Earth. What is the radius of the planet if the radius of Earth is 6.4×10^{6} m?

Think about

19. A fellow student proposes that the attraction between the Earth and the Moon is electrical rather than gravitational. How would you argue against this student's proposal?

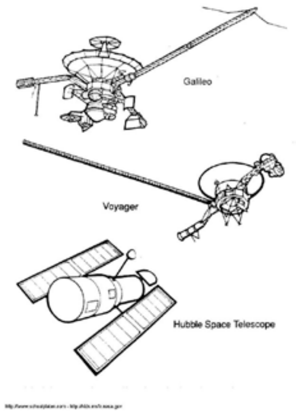
20. How does the force of attraction of the Earth on the Sun compare with:
a) the force of attraction of the Earth on the Moon?
b) the force of attraction of the Moon on the Sun?
c) the force of attraction of the Sun on the Earth?

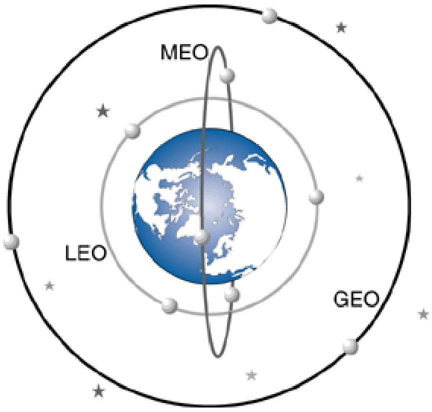
Answers

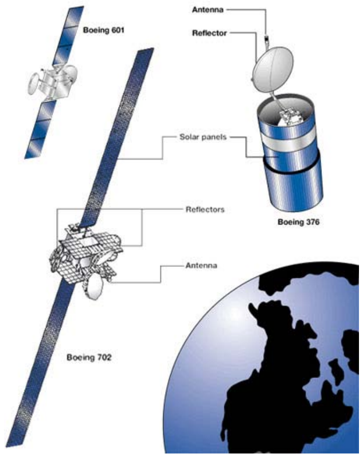
1. 2.67×10^{-6} N
2. a) 2.0×10^{20} N b) 1.0×10^{-47} N
3. 6.72×10^{-11}
4. 1.7×10^{3} N
5. 88 kg
6. 1770 km (Hint: don't forget to subtract the radius of the Earth.)
7. a) $\frac{F}{4}$ b) 36 F c) $\frac{4F}{9}$
8. 9.77 ms^{-2}
9. a) 9.47 ms^{-2} b) 2.7×10^{-3} ms^{-2}
10. 32g = 314 ms^{-2}
11. 2400 km
12. 600 N down
13. 13 612 N down
14. B
15. B
16. 3.84×10^{7} m from moon
17. 6.65×10^{-11} N m^{2} kg^{-2}
18. 4.3×10^{6} m (Hint: Assume planet and Earth are spherical)

Satellites

The centripetal force needed to keep a satellite in orbit is provided by gravitational attraction.





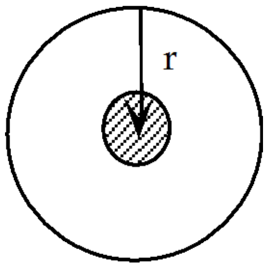


For a satellite of mass m travelling in a circular orbit at speed v and radius r:

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r}$$



Note that the radius of orbit is taken from the centre of the Earth, not the Earth's surface.

You must take care with this, as some questions will give you a satellite's altitude; then you would need to add the Earth's radius to the given altitude to find the radius of orbit.

From the equation, it should be noticed that the square of the velocity of a satellite is inversely proportional to its radius.

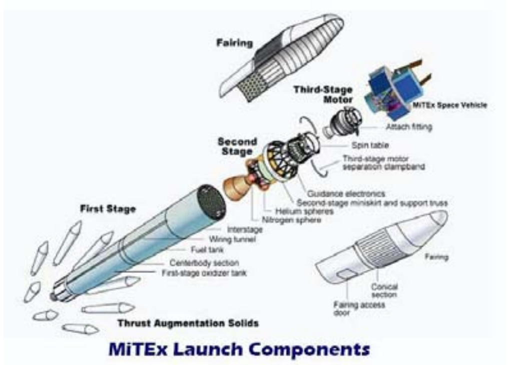
Thus a satellite in a lower orbit will have a higher speed than a satellite in a higher orbit.

To achieve a greater radius of orbit, a spacecraft must fire its rockets in a forward direction.

This will result in the spacecraft moving to a higher orbit, but moving with a reduced speed.

There is a reduction in kinetic energy but a greater increase in the spacecraft's potential energy.

Overall the total energy of a satellite in an outer orbit is greater than that in an inner orbit, even though its speed is slower



Example 1: At what velocity does a satellite orbit the Earth if it is 9.00 x 10^6 m from the Earth's centre? (Mass of Earth = 6.00 x 10^{24} kg).

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v^2 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{9 \times 10^6}$$

Thus $\underline{v = 6.67 \times 10^3}$ ms^{-1}

Period of Orbit (and Kepler's Laws)

The time taken for a satellite to complete one orbit is termed its period of revolution 'T'.

$$\boxed{\text{velocity} = \frac{2\pi r}{T}}$$

Since $v^2 = \frac{GM}{r}$ and $v = \frac{2\pi r}{T}$

then $\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$

$$\therefore \; T^2 = \frac{4\pi^2 r^3}{GM_e}$$

(Kepler's law $T^2 = kr^3$)

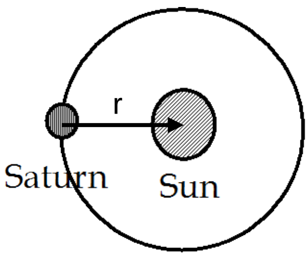
Example 2: Saturn has a sidereal period of 1.08 x 10^4 days and a radius of orbit of 1.43 x 10^9 km. Calculate the mass of the Sun based on this information.
[Note: Sidereal time is judged by reference to movement of the stars – not the sun]

T = 1.08 x 10^4 days

T = 9.33 x 10^8 s

r = 1.43 x 10^9 km

r = 1.43 x 10^{12} m



$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r} \quad and \quad v = \frac{2\pi r}{T}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\mathbf{M} = \frac{4\pi^2 r^3}{GT^2}$$

$$\mathbf{M} = \frac{4\pi^2(1.43 \times 10^{12})^3}{6.67 \times 10^{-11} \times (9.33 \times 10^8)^2}$$

$$\mathbf{M = 1.99 \times 10^{30} \; kg}$$

Geostationary (or Geosynchronous) Orbit

When a satellite is placed in orbit such that it remains above one position on the Earth's surface it is said to be in a geostationary orbit.

For a geostationary orbit about the Earth, the satellite must:

- be positioned above the equator (Why?)
- have a period of revolution equal to the period of the earth's rotation - one sidereal day (24h)
- revolve in the direction of the earth's rotation - easterly.

Example 3: Determine the radius for a geostationary orbit about the Earth. ($M_E = 6.00 \times 10^{24}$ kg)

T = 24.0 h

= 86400 s

M = 6.00×10^{24} kg

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r} \quad and \quad v = \frac{2\pi r}{T}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\therefore\ r^3 = \frac{GMT^2}{4\pi^2} = \frac{(6.67 \times 10^{-11}) \times (6.00 \times 10^{24}) \times (8.64 \times 10^4)^2}{4\pi^2}$$

$$\therefore\ \underline{r = 4.23 \times 10^7}\ \text{m}$$

Apparent Weightlessness in Orbit

In an orbiting space shuttle, the acceleration necessary to keep an astronaut and the space craft in orbit is provided by gravity.

Thus the astronaut and space shuttle both experience the same acceleration towards the Earth.

The astronaut and space shuttle are in a continuous state of free fall.

As a result, there appears to be no net force acing on the astronaut and so he experiences an apparent weightlessness.

The same phenomenon is observed by a person in a free falling lift.

Since the lift and occupant accelerate down at the same rate, the occupant would experience apparent weightlessness, even though he is being accelerated due to gravity.

EXERCISE 6: Satellite Motion

Solar System Data: Universal Gravitational Constant G = 6.67×10^{-11} N m^2 kg^{-2}

Body	Mass (kg)	Average radius (km)	'g ' at surface (ms^{-2})	Sidereal period (days)	Radius of orbit (km)
Moon	7.35×10^{22}	1738	1.62	27.3	3.8×10^{5}
Sun	1.97×10^{30}	695000	274		
Mercury	3.28×10^{23}	2570	3.9	88	5.8×10^{7}
Venus	4.82×10^{24}	6310	8.9	245	1.08×10^{8}
Earth	5.98×10^{24}	6370	9.80	365.26	1.50×10^{8}
Mars	6.37×10^{23}	3430	3.8	687	2.28×10^{8}
Jupiter	1.88×10^{27}	71800	26	4333	7.78×10^{8}
Saturn	5.62×10^{26}	60300	11	1.08×10^{4}	1.43×10^{9}
Uranus	8.62×10^{25}	26700	10	3.07×10^{4}	2.87×10^{9}
Neptune	1.0×10^{26}	24900	14	6.02×10^{4}	4.5×10^{9}

1. Find the gravitational attraction between two identical 40.0 kg lead spheres with centres 0.60 m apart. Compare this force with the Earth's gravitational force on one of the spheres.

2. Calculate the attractive force of the earth for the Moon.
How many times greater is the attraction of the Earth for the Sun?

3. When a lead sphere is placed 0.20 m from another lead sphere of mass 5.00 kg, the attraction of one for the other is found to be 7.00×10^{-8} N. What is the mass of the first sphere?

4. From the masses and radii given in the table (at the beginning of the questions), estimate the acceleration due to gravity on the surface of
a) the moon b) Venus c) Jupiter d) Mars.
What would be the weight of a 2.00 kg mass at the surface of each?

5. It is proposed to put a space station in a circular orbit at a distance of $\frac{1}{3}$ the Earth's radius above the Earth's **surface**. a) Find the acceleration due to gravity at this elevation?
b) Find the speed of the station if it is to go around the centre of the earth in a circular orbit.
c) How long will it take to make one complete revolution?

6. If a planet had a circular orbit about our sun with a radius 5.00 times that of the earth's orbit, what would its period of revolution and orbital speed be?

7. Find the radius of the orbit of a satellite which revolves about the earth in one sidereal day (86 164s). What is its orbital velocity?

8. Find the speed of a spacecraft circling the moon in a parking orbit at a radius of 1.80 x 10^3 km (60.0 km above the lunar surface). What is the period (time for one complete orbit) for this spacecraft?

9. A satellite with a mass of 1.50 x 10^3 kg is to be placed in a circular orbit of radius 8.00 x 10^3 km about the Earth's centre. Find the speed of the satellite in orbit.

10. A rocket is 1911 km (0.30 earth radius) above the surface of the Earth.
a) What is 'g 'at this height? b) Find the weight of a 5 kg mass at this point.

11. Sputnik, the first artificial earth satellite launched by the USSR on Oct 4, 1957, took 96.0 min. to traverse its almost circular orbit. Calculate its approximate height above the Earth's surface and the acceleration due to gravity at that height.

12. Halley's comet has been observed on every passage near the sun at intervals of from 74.0 to 79.0 years since 240 BC. (The variation in the period is due to perturbations produced by the major planets). Assuming that it describes an elliptical path about the sun with a 76.0 year period, calculate the approx. semi-major axis ('radius') of its orbit.

13. Show that if a body describes a circular path under the influence of the gravitational inverse square force law, the square of the period is proportional to the cube of the path radius (Kepler's third law for a circular orbit) and find the proportionality constant in terms of the central mass M and universal constants.

14. How fast must an aircraft be able to fly due west to keep the sun always on the meridian
a) at the equator and b) at 40.0º latitude? What is the centripetal acceleration in each case?

15. Imagine a planet having 3.00 times the average density of the earth and twice the radius. Calculate the acceleration due to gravity at the surface of this planet.

16. a) What is meant by saying a satellite is in a 'geosynchronous' orbit of the Earth?
b) Show that a satellite in such an orbit must be about 35 850 kilometres above the Earth's surface.
c) If a geosynchronous satellite is in orbit around the Earth at some latitude other that 0º, describe its motion.

Answers

1. 2.96 x 10^{-7} N; 7.55 x 10^{-10} N
2. 2.03 x 10^{20} N, 172 times
3. 8.40 g
4. 1.62 ms^{-2} ; 8.07 ms^{-2}; 24.3 ms^{-2};
5. a) 5.51 ms^{-2} b) 6840 ms^{-1} c) 7.80 x 10^3 s
6. 4084 days; 1.34 x 10^4 ms^{-1}
7. 4.22 x 10^7 m; 3.08 kms^{-1}
8. 1.65 x 10^3 ms^{-1} ; 6.85 x 10^3 s
9. 7.07 x 10^3 ms^{-1}
10. 5.80 ms^{-2} ; b) 29.0 N
11. 577 km; 8.27 ms^{-2}
12. 2.70 x 10^9 km
14. a) 1670 kmh^{-1}, 0.034 ms^{-2} b) 1280 kmh^{-1} , 0.026 ms^{-2}
15. 58.8 ms^{-2}